Calculus I - Exam 22-12-10

1. (1 Point)

For which x is the following series absolutely convergent?

$$\sum_{n=0}^{\infty}(n^2+2)2^n(-1)^n\frac{1}{(x+7)^n}$$

2. (1 Point)

Prove that the sequence x_n, defined by:

$$x_{n+1} = x_n^2 - 10x_n + 30 \text{ , and } x_0 = 5.3$$

Is monotonically decreasing and bounded from below.
Is x_n convergent? If so, then find lim_{x_n}.

3. (3 Points)

Find the following limits:

$$\lim_{x\to\infty} = \frac{2x^8 - 12x + 6}{x^8 - 12x^2 + 8}$$

$$\lim_{x\to 9} = \frac{\sqrt{x-8}-1}{x^2-81}$$

$$\lim_{t\to 1} = \frac{ln(sin\frac{t\pi}{2})}{(t-1)^2}$$

4. (1 Point)

Consider the function $f(x)$:

$$f(x) = \begin{cases} \frac{x^3-9x}{x^2-3x}, & \text{if } x \neq 3 \text{ and } x > 0 \\ 11, & \text{if } x = 3 \end{cases}$$

Is $f(x)$ continuous at every $a \in (0, +\infty)$?
If $f(x)$ is not continuous, then can you modify the value of $f(x)$ at $x = 3$, so that the modified function becomes continuous on $(0, +\infty)$?

5. (1 Point)

Using the definition of limit, prove that:

$$\lim_{x\to 5}\frac{x}{x+4} = \frac{5}{9}$$

6. (1 Point)

Consider the function $f(x) = 2x^2e^{-x^2} + 3$.
Find the local and global max. and min. points of $f(x)$, and determine that intervals on which the function $f(x)$ is increasing or decreasing.

7. (1/2 Point)

Compute the following integrals:

$$\int 2e^{4x}sin(e^{4x})dx$$

8. (1 Point)

Prove using the definition of the limit of a sequence, that:

$$\lim_{n\to\infty}\frac{3\sqrt{n}}{\sqrt{n}+2} = 3$$

9. (1 Point)

Let $f(x) = \sqrt{2+x^2}$ Using the definition of the derivative prove that $f(x)$ is differentiable and find its derivative $f'(x)$.