

Testexam Calculus 2, May 2009

1. Let $f(x, y) = e^{xy}$, $x(u, v) = 3u \sin v$ and $y(u, v) = 4v^2u$. Give the partial derivatives $\frac{\partial g}{\partial u}$ and $\frac{\partial g}{\partial v}$, where the function g is defined by $g(u, v) = f(x(u, v), y(u, v))$.
2. Let $f(x, y) = xe^{x^2-y^2}$. Give an equation for the tangent plane at $(1, 1, 1)$.
3. The curve K is given by the parametrization

$$\begin{cases} x(t) &= t^3 - t \\ y(t) &= t^4 - 5t^2 + 4 \end{cases} \quad t \in \mathbb{R}$$

- (a) The curve K intersects itself at one point. Calculate this point.
 - (b) At which point(s) does the curve have a horizontal tangent line?
At which point(s) does the curve have a vertical tangent line?
 - (c) Calculate the area enclosed by K .
4. Let R be the triangle enclosed by the lines $y = x$, $y = 0$ and $x = 4$. Calculate $\iint_R 4e^x - 5 \sin y \, dA$ in two different ways, i.e. with two different iterated integrals.
5. Calculate $\int_{y=0}^{y=1} \int_{x=y}^{x=1} e^{x^2} dx dy$.
6. Let $f(x) = 9 - x^2$, $x \in [0, 3]$. Determine the volume of the solid obtained by rotating f about the y -axis in two ways.
 - (a) With the method of cylindrical shells.
 - (b) With the method of slices, perpendicular to the y -axis.
7. Consider the differential equation given by

$$yy' + yy'' + (y')^2 = 0.$$

- (a) Let $z = y^2$ and derive a differential equation for z .
 - (b) Give all solutions for the differential equation you derived in a).
Then give all solutions for the original differential equation.