

CALCULUS KEN1440

RESIT EXAMINATION

Examiners: Carlo Galuzzi
Date: June 27, 2016
Time: 14:00-17:00
Place: Tapijn, Building Z

Notes:

- 1) This exam is a closed-book exam.
- 2) Use of calculators is forbidden.
- 3) You can use only formulas that will be given to you.
- 4) You have 180 minutes for this exam.
- 5) The exam consists of 4 pages (including this page and appendix).
- 6) Always explain your answers.
- 7) The number of points for each question is given (in bold).
- 8) The maximum number of points is **100**.
- 9) Before answering the questions, please first read all exam questions, and then make a plan to spend the available time.
- 10) When answering the questions please do not forget:
 - to write your name and student number on each answer page;
 - to number the answers; and
 - to number the answer pages.
- 11) When you are asked to calculate something, it is asked for both **the correct solution** and **the procedure to identify it**, not only a number.

GOOD LUCK!!!

EXERCISE 1 [24 POINTS]

Compute the following limits, if they exist, or explain why they do not exist:

- a. **(2 points)** $\lim_{x \rightarrow \infty} \frac{2^x + x^2}{e^x + 1}$
- b. **(2 points)** $\lim_{x \rightarrow \infty} \frac{x^6 + \cos^8 x}{e^x}$
- c. **(4 points)** $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x^2}\right)^x$
- d. **(4 points)** $\lim_{x \rightarrow \pm\infty} \frac{10^x + x^2}{2x^2 + e^x}$
- e. **(4 points)** $\lim_{x \rightarrow \pm\infty} \frac{|x^2 + x + 3|}{\sqrt{x^4 + 12x + 1}}$
- f. **(4 points)** $\lim_{x \rightarrow 0} \frac{\sin(5x^2 + 3x)}{\sqrt{x^2 + x}}$
- g. **(4 points)** $\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 2} - \sqrt{2}}{x}$

EXERCISE 2 [24 POINTS]

Let $f(x) = \sqrt{\frac{1}{x^2 + 2}}$

- a. **(3 points)** Determine the domain of f . Is f continuous on its domain? Why?
- b. **(4 points)** Compute the first derivative of f . Determine from this derivative for which values of x the function f is increasing and decreasing. Does it have local minimum(s), maximum(s)? If so, at which values of x ?
- c. **(4 points)** Compute the second derivative of f . Determine from this derivative for which values of x the function f is convex (=concave up) and concave (=concave down). Does it have inflection points? If so, at which values of x ?
- d. **(3 points)** Does f have vertical, horizontal, and/or oblique asymptotes? Use the limits of f to show your answer.
- e. **(3 points)** Is f even or odd? Why?
- f. **(2 points)** Find the point(s) at which the graph of f intersects the x -axis and the y -axis, if there are such point(s).
- g. **(2 points)** Find when $f(x) \geq 0$.
- h. **(3 points)** Based on your previous answers (a-g), sketch the graph of the function f . Show all the properties in (a-g) in this graph.

EXERCISE 3 [18 POINTS]

Compute the following derivatives:

- a. **(3 points)** $\left(\sqrt{\frac{1}{\cos(e^{2x})}}\right)'$
- b. **(3 points)** $\left(a^{x^2+3} \tan(x^2)\right)'$
- c. **(4 points)** $\left(e^{2x^2+3} \cos x\right)''$
- d. **(4 points)** $\left(\ln \frac{1}{\cos(x^2)}\right)''$
- e. **(4 points)** $\left((x+1)(x+2)(x+3)\right)''$

EXERCISE 4 [19 POINTS]

Compute the following integrals:

- a. **(3 points)** $\int \cos^3 x \, dx$
- b. **(4 points)** $\int x \cos(2x+2) \, dx$
- c. **(5 points)** $\int \frac{x^3}{x^2+2x+1} \, dx$
- d. **(3 points)** $\int \frac{1}{2x(x^2+6)} \, dx$
- e. **(4 points)** $\int_1^{10} \frac{1}{\sqrt[3]{x-2}} \, dx$

EXERCISE 5 [15 POINTS]

- a. **(5 points)** Define what is a continuous extension of a function and what is a removable discontinuity. Give an example.
- b. **(5 points)** State the max-min theorem for continuous functions and give an example.
- c. **(5 points)** State the fundamental theorem of calculus (both part 1 and part 2).

APPENDIX - EXAM FORMULAS

Some important derivatives

$f(x)$	$f'(x)$	remark
$c \in \mathbb{R}$	0	
x^m	$m x^{m-1}$	$m \in \mathbb{R}$
$ x $	$\frac{x}{ x } = \operatorname{sgn} x$	
$\sin x$	$\cos x$	
$\cos x$	$-\sin x$	
$\tan x$	$\sec^2 x$	
$\sec x$	$\sec x \tan x$	
$\cot x$	$-\csc^2 x$	
$\csc x$	$-\csc x \cot x$	
e^x	e^x	
$\ln x$	$\frac{1}{x}$	$x > 0$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$	
$\arctan x$	$\frac{1}{1+x^2}$	
$\tan x + c$	$\sec^2 x$	
$\cot x + c$	$-\csc^2 x$	
$\sec x + c$	$\sec x \tan x$	
$\csc x + c$	$-\csc x \cot x$	

Some important indefinite integrals (with $c \in \mathbb{R}$)

$f(x)$	$\int f(x) dx$	remark
1	$x + c$	$\int dx = \int 1 dx = x + c$
x^m	$\frac{x^{m+1}}{m+1} + c$	$m \in \mathbb{R} \setminus \{-1\}$
$\sqrt{x}$	$\frac{2}{3} x^{3/2} + c$	$x > 0$
$\frac{1}{x}$	$\ln x + c$	$x \neq 0$
e^x	$e^x + c$	
$\sin x$	$-\cos x + c$	
$\cos x$	$\sin x + c$	
$\sec^2 x$	$\tan x + c$	$x \neq (2k+1)\frac{\pi}{2}$
$\csc^2 x$	$-\cot x + c$	$x \neq k\pi$
$\sec x \tan x$	$\sec x + c$	$x \neq (2k+1)\frac{\pi}{2}$
$\csc x \cot x$	$-\csc x + c$	$x \neq k\pi$
$\frac{1}{\sqrt{1-x^2}}$	$\arcsin x + c$	$x \in (-1, 1)$
$\frac{1}{1+x^2}$	$\arctan x + c$	

Some important trigonometric identities

$\sin^2 x + \cos^2 x = 1$
$1 + \tan^2 x = \sec^2 x$
$1 + \cot^2 x = \csc^2 x$
$\sin(2x) = 2 \sin x \cos x$
$\cos(2x) = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$
$\sin^2 x = \frac{1 - \cos(2x)}{2}$
$\cos^2 x = \frac{1 + \cos(2x)}{2}$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$