CALCULUS KEN1440
TRIAL FINAL EXAMINATION

Examiners:	Carlo Galuzzi
Date:	March 2016
Time:	**-**
Place:	****

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Notes:

1) This exam is a closed-book exam.
2) Use of calculators is forbidden.
3) You can use only formulas that will be given to you.
4) You have 180 minutes for this exam.
5) The exam consists of 4 pages (including this page and appendix).
6) Always explain your answers.
7) The number of points for each question is given (in bold).
8) The maximum number of points is **100**.
9) Before answering the questions, please first read all exam questions, and then make a plan to spend the available time.
10) When answering the questions please do not forget:
    - to write your name and student number on each answer page;
    - to number the answers; and
    - to number the answer pages.
11) When you are asked to calculate something, it is asked for both **the correct solution** and **the procedure to identify it**, not only a number.

GOOD LUCK!!!

EXERCISE 1 [18 POINTS]

Compute the following limits, if they exist, or explain why they do not exist:

a. (**4 points**) $\lim\limits_{x\to\infty} (\sqrt{x^2+3\,x} - x)$

b. (**4 points**) $\lim\limits_{x\to 0} \frac{\sin(x)-\sin(2x)}{e^x-1-2\,x-x^2}$

c. (**3 points**) $\lim\limits_{x\to 12} \frac{x^2+1}{12-x}$

d. (**4 points**) $\lim\limits_{x\to\sqrt{2}} \frac{|x-\sqrt{2}|}{x^2-2}$

e. (**3 points**) $\lim\limits_{x\to 0} \frac{2x^2}{e^x-1}$

EXERCISE 2 [22 POINTS]

Let $f(x) = \frac{x+1}{x-1}$

a. (**3 points**) Determine the domain of f. Is f continuous on its domain? Why?

b. (**4 points**) Compute the first derivative of f. Determine from this derivative for which values of x the function f is increasing and decreasing. Does it have local minimum(s), maximum(s)? If so, at which values of x?

c. (**4 points**) Compute the second derivative of f. Determine from this derivative for which values of x the function f is convex (=concave up) and concave (=concave down). Does it have inflection points? If so, at which values of x?

d. (**3 points**) Does f have vertical, horizontal, and/or oblique asymptotes? Use the limits of f to show your answer.

e. (**3 points**) Is f even or odd? Why?

f. (**2 points**) Find the point(s) at which the graph of f intersects the x-axis and the y-axis, if there are such point(s).

g. (**3 points**) Based on your previous answers (a-f), sketch the graph of the function f. Show all the properties in (a-f) in this graph.

EXERCISE 3 [18 POINTS]

Compute the following derivatives:

a. (**4 points**) $(8^{x\ \cos\, x})'$

b. (**3 points**) $\left(\frac{\ln(x)}{x}\right)'$

c. (**3 points**) $(|x-1|)'$

d. (**4 points**) $(x\ \ln(x)\ \sin(x))'$

e. (**4 points**) $\left(\frac{2x}{x^2-4}\right)''$

EXERCISE 4 [18 POINTS]

Compute the following integrals:

a. (**4 points**) $\int \arcsin(x)\,\mathrm{d}x$

b. (**5 points**) $\int \frac{x^2+3\,x+1}{x^2-1}\,\mathrm{d}x$

c. (**5 points**) $\int\limits_0^2 \sqrt{4-x^2}\,\mathrm{d}x$. Use the substitution $x = 2\sin\theta$ with $-\pi/2 \le \theta \le \pi/2$.

d. (**4 points**) $\int\limits_0^1 e^x\sqrt{1+e^x}\mathrm{d}x$

EXERCISE 5 [24 POINTS]

a. (**6 points**) List the properties of the **finite** limits (a.k.a. **limits laws**).

b. (**6 points**) State the **Squeeze (or Sandwich) theorem** for functions and explain it with an example.

c. (**6 points**) State the **Mean-Value Theorem** for functions and explain it with an example.

d. (**6 points**) State both **l'Hôpital's rules** and show an example for both of them.

APPENDIX - EXAM FORMULAS

Some important derivatives

$f(x)$	$f'(x)$	remark
$c \in \mathbb{R}$	0	
x^m	$m\,x^{m-1}$	$m \in \mathbb{R}$
$\lvert x\rvert$	$\frac{x}{\lvert x\rvert} = \operatorname{sgn} x$	
$\sin\ x$	$\cos\ x$	
$\cos\ x$	$-\sin\ x$	
$\tan\ x$	$\sec^2\ x$	
$\sec\ x$	$\sec\ x \tan\ x$	
$\cot\ x$	$-\csc^2\ x$	
$\csc\ x$	$-\csc\ x\ \cot\ x$	
e^x	e^x	
$\ln\ x$	$\frac{1}{x}$	$x > 0$
$\arcsin\ x$	$\frac{1}{\sqrt{1-x^2}}$	
$\arctan\ x$	$\frac{1}{1+x^2}$	
$\tan\ x + c$	$\sec^2\ x$	
$\cot\ x + c$	$-\csc^2\ x$	
$\sec\ x + c$	$\sec\ x\ \tan\ x$	
$\csc\ x + c$	$-\csc\ x\ \cot\ x$	

Some important indefinite integrals (with $c \in \mathbb{R}$)

$f(x)$	$\int f(x)\,\mathrm{d}x$	remark
1	$x + c$	$\int \mathrm{d}x = \int 1\,\mathrm{d}x = x + c$
x^m	$\frac{x^{m+1}}{m+1} + c$	$m \in \mathbb{R} \setminus \{-1\}$
$\sqrt{x}$	$\frac{2}{3}x^{3/2} + c$	$x > 0$
$\frac{1}{x}$	$\ln\ \lvert x\rvert + c$	$x \neq 0$
e^x	$e^x + c$	
$\sin\ x$	$-\cos\ x + c$	
$\cos\ x$	$\sin\ x + c$	
$\sec^2\ x$	$\tan\ x + c$	$x \neq (2\,k+1)\frac{\pi}{2}$
$\csc^2\ x$	$-\cot\ x + c$	$x \neq k\,\pi$
$\sec\ x\ \tan\ x$	$\sec\ x + c$	$x \neq (2\,k+1)\frac{\pi}{2}$
$\csc\ x\ \cot\ x$	$-\csc\ x + c$	$x \neq k\,\pi$
$\frac{1}{\sqrt{1-x^2}}$	$\arcsin\ x + c$	$x \in (-1, 1)$
$\frac{1}{1+x^2}$	$\arctan\ x + c$	

Some important trigonometric identities

$\sin^2 x + \cos^2 x = 1$
$1 + \tan^2 x = \sec^2 x$
$1 + \cot^2 x = \csc^2 x$
$\sin(2\,x) = 2\ \sin\ x\ \cos\ x$
$\cos(2\,x) = 2\ \cos^2\ x - 1 = 1 - 2\ \sin^2 x$
$\sin^2 x = \frac{1-\cos(2\,x)}{2}$
$\cos^2 x = \frac{1+\cos(2\,x)}{2}$
$\sin(x \pm y) = \sin\ x \cos\ y \pm \cos\ x \sin\ y$