a) $\lim\limits_{x \to 0} \dfrac{\tan 10x}{x\sqrt{3}} = \dfrac{0}{0} \Rightarrow$ L'Hopital

$= \lim\limits_{x \to 0} \dfrac{\frac{1}{1+100x^2} \cdot 10}{\sqrt{3}} = \dfrac{10}{\sqrt{3}}$

b) $\lim\limits_{x \to +\infty} \dfrac{x^7 - 5\sin x}{2x^2 + 7x^7}$ $\quad |\sin x| \leq 1 \Rightarrow -\sin x \leq 1$

$\lim\limits_{x \to +\infty} \dfrac{x^7 - 5(-1)}{2x^2 + 7x^7} \leq \lim\limits_{x \to +\infty} \dfrac{x^7 - 5\sin x}{7x^7 + 2x^2} \leq \lim\limits_{x \to +\infty} \dfrac{x^7 - 5(1)}{7x^7 + 2x^2}$

$= \dfrac{1}{7}$ $\quad \Rightarrow \quad \hookrightarrow = \dfrac{1}{7}$ $\quad = \dfrac{1}{7}$

c) $\lim\limits_{x \to +\infty} \left(1 + \dfrac{2}{x}\right)^{3x} = \lim\limits_{x \to +\infty} e^{\ln\left(1+\frac{2}{x}\right)^{3x}}$

$\Rightarrow \lim\limits_{x \to +\infty} \ln\left(1 + \dfrac{2}{x}\right)^{3x} = \lim\limits_{x \to +\infty} 3x \ln\left(1 + \dfrac{2}{x}\right) = \infty \cdot 0$

$\Rightarrow \quad = \lim\limits_{x \to +\infty} \dfrac{\ln\left(1+\frac{2}{x}\right)}{\frac{1}{3x}} = \dfrac{0}{0} \Rightarrow$ L'hopital

$= \lim\limits_{x \to +\infty} \dfrac{\frac{1}{1+\frac{2}{x}} \cdot \left(-\frac{2}{\cancel{x^2}}\right)}{\frac{1}{3}\left(-\frac{1}{\cancel{x^2}}\right)} = 6$

$\Rightarrow \lim\limits_{x \to +\infty} \left(1 + \dfrac{2}{x}\right)^{3x} = e^6$

1-2

d) $\lim_{x \to +\infty} \frac{|x-2|}{x^3+x-59} = 0$ degree N < degree D

e) $\lim_{x \to +\infty} \frac{5x^7+6x^3+2}{e^{6x}} = 0$ N = polynomial function
D = exponential function

f) $\lim_{x \to 0} \frac{10^x - 8^x}{3x} = \frac{0}{0} \Rightarrow$ L'Hopital

$$= \lim_{x \to 0} \frac{10^x \cdot \ln 10 - 8^x \ln 8}{3} = \frac{1}{3}(\ln 10 - \ln 8)$$

$$= \frac{1}{3} \ln \frac{5}{4}$$

g) $\lim_{x \to 0^+} (\sin x)^{\tan x} = \lim_{x \to 0^+} e^{\ln (\sin x)^{\tan x}}$

$$\Rightarrow \lim_{x \to 0^+} \ln (\sin x)^{\tan x} = \lim_{x \to 0^+} \tan x \cdot \ln(\sin x)$$

$$= \lim_{x \to 0^+} \frac{\ln(\sin x)}{\frac{1}{\tan x}} = \frac{\infty}{\infty} \Rightarrow \text{L'Hopital}$$

$$= \lim_{x \to 0^+} \frac{\frac{1}{\sin x} \cdot \cos x}{-\frac{1}{\tan^2 x}\left(\frac{1}{1+x^2}\right)} = \lim_{x \to 0^+} - \cancel{\frac{\cos x}{\sin x}} \cdot \frac{\sin^2 x}{\cos^2 x}(1+x^2) = 0$$

$$\Rightarrow \lim_{x \to 0^+} (\sin x)^{\tan x} = 1$$

$$\lim_{x\to 0} \frac{\sqrt{x+57}-\sqrt{57}}{x} = \lim_{x\to 0} \frac{\sqrt{x+57}-\sqrt{57}}{x} \cdot \frac{\sqrt{x+57}+\sqrt{57}}{\sqrt{x+57}+\sqrt{57}}$$

$$= \lim_{x\to 0} \frac{x+\cancel{57}-\cancel{57}}{x\left(\sqrt{x+57}+\sqrt{57}\right)} = \frac{1}{2\sqrt{57}}$$

$$f(x) = \frac{x^2+2}{2x+1}$$

a) $D(f) = \mathbb{R} \setminus \{-\frac{1}{2}\}$

b) $$f'(x) = \frac{2x(2x+1) - (x^2+2)\cdot 2}{(2x+1)^2}$$

$$= \frac{4x^2+2x-2x^2-4}{(2x+1)^2} = \frac{2x^2+2x-4}{(2x+1)^2} = 2\cdot\frac{x^2+x-2}{(2x+1)^2}$$

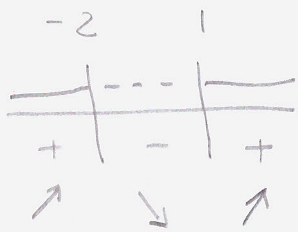
Roots of x^2+x-2 : $\frac{-1\pm\sqrt{1+8}}{2} = \frac{-1\pm 3}{2} = \left< \begin{matrix} 1 \\ -2 \end{matrix}\right.$

$\rightarrow \quad f'(x) = 2\cdot\frac{(x-1)(x+2)}{(2x+1)^2}$

$f'(x) = 0 \quad \Rightarrow \quad \begin{matrix} x = 1 \\ x = -2 \end{matrix}$

$f'(x) > 0 \quad \Rightarrow \quad \begin{matrix} x > 1 \\ x < -2 \end{matrix}$

$(2x+1)^2 > 0$ always



$\Rightarrow \quad x = -2 \quad \max \quad f(-2) = -2$

$\quad\;\; x = 1 \quad \min \quad f(1) = 1$

c) $$f''(x) = 2\cdot\frac{(2x+1)(2x+1)^2 - (x^2+x-2)\cdot 2(2x+1)\cdot 2}{(2x+1)^4}$$

2-2

$$= 2 \cdot \frac{(2x+1)(4x^2+1+4x) - 4(2x+1)(x^2+x-2)}{(2x+1)^4}$$

$$= 2 \cdot \frac{\cancel{8x^3} + 2x + \cancel{8x^2} + \cancel{4x^2} + 1 + 4x - \cancel{8x^3} - \cancel{8x^2} + 16x - \cancel{4x^2} - 4x + 8}{(2x+1)^4}$$

$$2 \cdot \frac{18x+9}{(2x+1)^4} = 2 \cdot 9 \cdot \frac{2x+1}{(2x+1)^4} = \frac{18}{(2x+1)^3}$$

$f''(x) = 0$ never $\qquad f''(x) > 0 \Rightarrow x > -\frac{1}{2}$

$-\frac{1}{2}$

d) Vertical asymptote : $x = -\frac{1}{2}$

$\lim\limits_{x \to -\frac{1}{2}^+} f(x) = +\infty \qquad \lim\limits_{x \to -\frac{1}{2}^-} f(x) = -\infty$

Oblique asymptote : $\frac{x^2+2}{2x+1}$

$$\begin{array}{r|l} & \frac{1}{2}x - \frac{1}{4} \\ \hline 2x+1 & x^2+2 \\ & x^2 + \frac{1}{2}x \\ \hline & -\frac{1}{2}x + 2 \\ & -\frac{1}{2}x - \frac{1}{4} \\ \hline & 2 + \frac{1}{4} = \frac{9}{4} \end{array}$$

$$\Rightarrow \frac{x^2+2}{2x+1} = \underbrace{\frac{1}{2}x - \frac{1}{4}}_{\text{asymptote}} + \frac{9}{4}\,\frac{1}{x^2+2}$$

e) f neither odd nor even

f) $f(0) = 2$; $\frac{x^2+2}{2x+1} = 0 \Rightarrow x^2+2 = 0$ never on $\mathbb{R}$

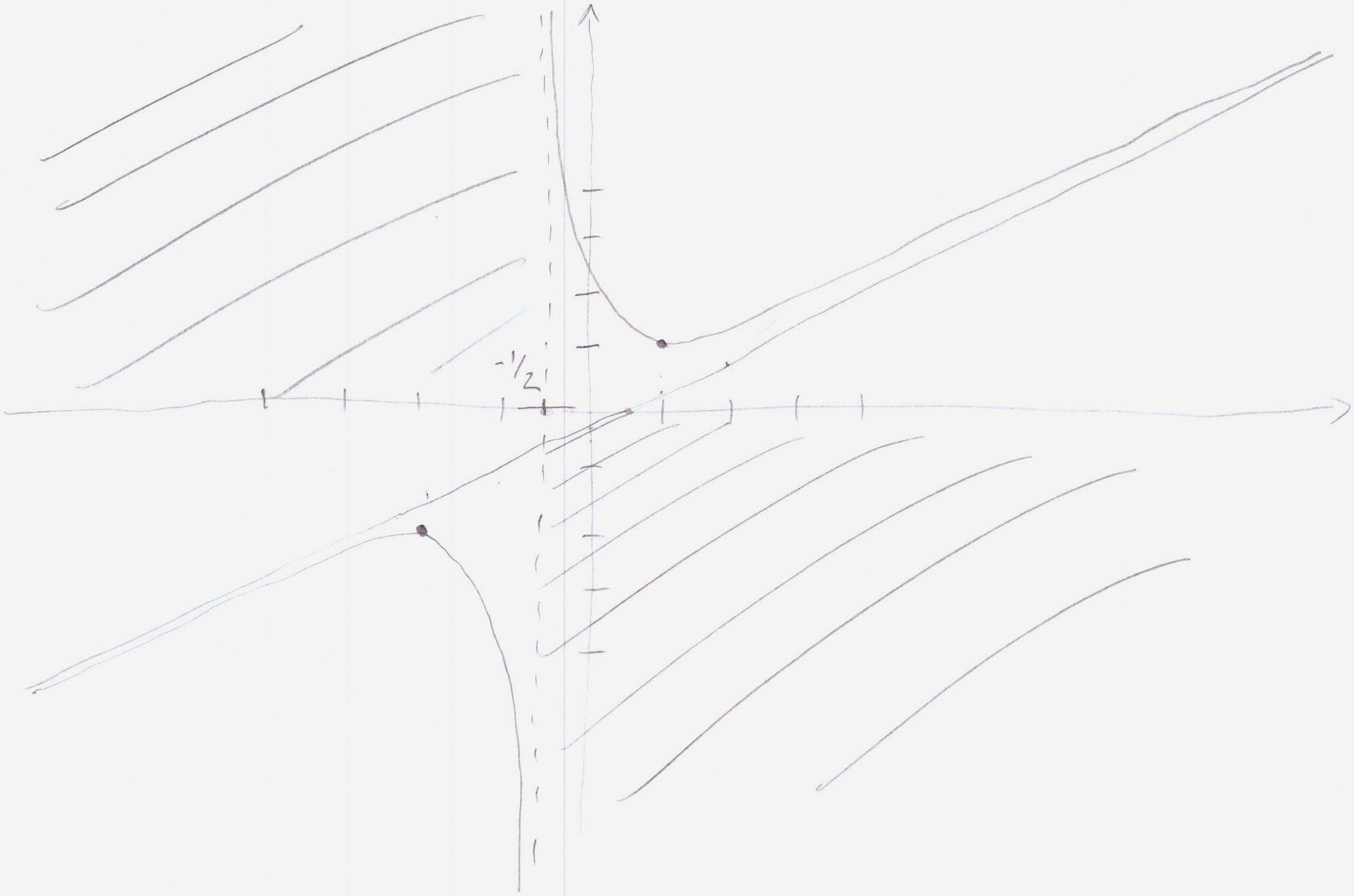
g) $f(x) > 0 \rightarrow x^2+2 > 0$ always

$2x+1 > 0 \Rightarrow x > -\frac{1}{2}$

$-\frac{1}{2}$

$f(x) < 0 \mid f(x) > 0$

2-3



a) $\left(\cos\sqrt{\sin\frac{1}{x}}\right)'$

$$= -\sin\sqrt{\sin\frac{1}{x}}\cdot\frac{1}{2}\left(\sin\frac{1}{x}\right)^{-1/2}\cdot\cos\frac{1}{x}\cdot\left(-\frac{1}{x^2}\right) =$$

$$= \frac{1}{2x^2}\ \frac{\sin\sqrt{\sin\frac{1}{x}}\ \cos\frac{1}{x}}{\sqrt{\sin\frac{1}{x}}}$$

b) $\left(e^{a^{x^2}}\tan\frac{1}{x^2}\right)' = e^{a^{x^2}}\cdot a^{x^2}\cdot 2x\cdot\ln a\ \tan\frac{1}{x^2} +$

$$+ e^{a^{x^2}}\ \frac{1}{1+\frac{1}{x^4}}\left(-\frac{2}{x^3}\right)$$

$$= e^{a^{x^2}}\left[2x\ln a\ a^{x^2}\tan\frac{1}{x^2} - \frac{2x}{x^4+1}\right]$$

c) $\left(\sqrt{\frac{1}{x^2+3}}\right)''$

$$\Rightarrow f'(x) = \frac{1}{2}\left(\frac{1}{x^2+3}\right)^{-\frac{1}{2}}\cdot\frac{-1}{(x^2+3)^2}\cdot 2x$$

$$= \frac{-x}{(x^2+3)^{3/2}}$$

$$\Rightarrow f''(x) = \frac{-(x^2+3)^{3/2} + x\,\frac{3}{2}(x^2+3)^{1/2}\cdot 2x}{(x^2+3)^3}$$

$$= \frac{-1}{(x^2+3)^{3/2}} + \frac{3x^2}{(x^2+3)^{5/2}}$$

$$= \frac{-(x^2+3)+3x^2}{(x^2+3)^{5/2}} = \frac{2x^2-3}{(x^2+3)^{5/2}}$$

3-2

d) $\left(x^3 e^{2x}\right)''$

$$f'(x) = 3x^2 e^{2x} + x^3 2e^{2x} = e^{2x}\left(2x^3+3x^2\right)$$

$$f''(x) = 2e^{2x}\left(2x^3+3x^2\right) + e^{2x}\left(6x^2+6x\right)$$

$$= e^{2x}\left(4x^3+6x^2+6x^2+6x\right)$$

$$= 2xe^{2x}\left(2x^2+6x+3\right)$$

e) $\left[\left(2x^2+1\right)\cos x\right]''$

$$f'(x) = 4x \cos x + \left(2x^2+1\right)(-\sin x)$$

$$= 4x\cos x - \left(2x^2+1\right)\sin x$$

$$f''(x) = 4\cos x - 4x\sin x - 4x\sin x - \left(2x^2+1\right)\cos x$$

$$= 3\cos x - 8x\sin x - 2x^2\cos x$$

a) $\int \arccos x \, dx$

$\arccos x \qquad x$

$-\frac{1}{\sqrt{1-x^2}} \qquad dx$

$= x \arccos x + \int \frac{x}{\sqrt{1-x^2}} dx =$

$= x \arccos x - \frac{1}{2} \int \frac{-2x}{\sqrt{1-x^2}} dx$

$= x \arccos x - \cancel{\frac{1}{2}}^{-1} \frac{(1-x^2)^{1/2}}{+\cancel{1/2}} + C = x \arccos x - \sqrt{1-x^2} + C$

b) $\int \frac{dx}{2x(x^2+6)} = \frac{1}{2} \int \frac{dx}{x(x^2+6)}$

$\frac{1}{x(x^2+6)} = \frac{A}{x} + \frac{Bx+C}{x^2+6} = \frac{Ax^2+6A+Bx^2+Cx}{x(x^2+6)}$

$A+B=0 \qquad\qquad B=-\frac{1}{6}$

$C=0 \qquad c=0$

$6A=1 \qquad A=\frac{1}{6}$

$\Rightarrow \quad = \frac{1}{2} \int \frac{1}{6}\frac{1}{x} - \frac{1}{6}\frac{x}{x^2+6} dx$

$= \frac{1}{12} \ln|x| - \frac{1}{24} \ln|x^2+6| + C$

(4-2)

c) $\displaystyle\int \frac{x^4}{\underbrace{x^2-2x-3}_{(x+1)(x-3)}}\,dx$

$$\begin{array}{r|l} & x^2+2x+7 \\ \hline x^2-2x-3 & x^4 \\ & x^4-2x^3-3x^2 \\ \hline & 2x^3+3x^2 \\ & 2x^3-4x^2+6x \\ \hline & 7x^2+6x \\ & 7x^2-14x-21 \\ \hline & +20x+21 \end{array}$$

$$= \int \underbrace{x^2+2x+7} + \frac{20x+21}{\underbrace{(x+1)(x-3)}}\,dx$$

$=$

$$\int x^2+2x+7\,dx = \frac{x^3}{3}+x^2+7x+C$$

$$\int \frac{20x+21}{(x+1)(x+3)}\,dx \qquad \frac{A}{x+1}+\frac{B}{x+3}$$

$$Ax-3A+Bx+B = 20x+21$$

$$A+B=20 \qquad B=20-A$$

$$-3A+B=21 \qquad -3A+20-A=21 \qquad -4A=+1$$

$$A=-\frac{1}{4} \qquad B=20+\frac{1}{4}=\frac{81}{4}$$

4-3

$$\Rightarrow \int \frac{20x+21}{(x+1)(x-3)}\,dx = -\frac{1}{4}\int \frac{1}{x+1}\,dx + \frac{81}{4}\int \frac{1}{x-3}\,dx$$

$$= -\frac{1}{4}\ln|x+1| + \frac{81}{4}\ln|x-3| + C_2$$

$$\Rightarrow \int \frac{x^4}{x^2-2x-3}\,dx = \frac{x^3}{3} + x^2 + 7x - \frac{1}{4}\ln|x+1| + \frac{81}{4}\ln|x-3| + C_3$$

d) $\int \sin 4x \cos 4x \, dx$

$$= \frac{1}{2}\int 2 \cdot \sin 4x \cos 4x \, dx$$

$$= \frac{1}{2}\int \sin 8x \, dx = -\frac{\cos 8x}{16} + C$$

e) $\int_3^{+\infty} \frac{dx}{x \ln^2 x}$ improper

$$= \lim_{c \to +\infty} \int_3^c \frac{dx}{x \ln^2 x}$$

$\ln x = t$

$\frac{1}{x}\,dx = dt$

$$= \lim_{c \to +\infty} \int_{\ln 3}^{\ln c} \frac{1}{t^2} = \lim_{c \to +\infty} \frac{-1}{t}\Big|_{\ln 3}^{\ln c}$$

$$= \lim_{c \to +\infty} \frac{-1}{\ln c} + \frac{1}{\ln 3} = \frac{1}{\ln 3}$$