

Exam

Logic
June 2004

This exam is NOT an open-book exam.
You will get an auxiliary sheet with the most important formulas.

Clarify all your answers sufficiently!

Indicate in semantic tableaux exactly which reduction rules you are using.

In the case of derivations, indicate exactly from which formula('s) each formula is derived and if the conditions are met (if applicable).

This exam contains 9 assignments. For each assignment will be indicated the maximum amount of points that you can obtain for a right answer. The exam mark is the number of points for all assignments together, divided by 10.

1. (10 points)

About the students Kees and Jan we know the following:

- i. If Kees passes his Logic exam, Jan will also pass his Logic exam.
- ii. If Jan passes his Logic exam, Kees will pass his Software Engineering exam.
- iii. Kees will pass his Logic exam or his Software Engineering exam, but not both.

Will Jan pass his Logic exam?

Will Kees pass his Software Engineering exam?

Determine the answer to these questions (yes or no) with the aid of a truth table (model elimination).

2. Semantic tableaux propositional logic. (12 points)

- a. Explain the $\rightarrow_R$ reduction rule.
- b. Given is the consequence $p \wedge (q \vee r) \ / \ (p \wedge q) \vee (p \wedge r)$. Investigate the validity of this consequence with the aid of a semantic tableau. If this consequence is not valid, give all the counterexamples.
- c. Investigate by means of a semantic tableau whether the formula $(p \rightarrow q) \vee (q \rightarrow p)$ is a tautology.

3. Prove by means of natural deduction (propositional logic) (12 points).

Prove by means of natural deduction:

- a. $(p \wedge q) \rightarrow r, s \rightarrow p, t \rightarrow q \vdash (s \wedge t) \rightarrow r$
- b. $p \vee q, p \rightarrow r, r \rightarrow s, q \rightarrow t \vdash s \vee t$
- c. $r \rightarrow \neg p \vdash \neg(p \wedge r)$

4. (10 points)

Translate the following formula into an equivalent formula in prenex form:

$$((\exists y R(x,y)) \rightarrow (\forall x S(x,y))) \rightarrow ((\exists x T(x,y)) \vee (\forall y U(x,y)))$$

5. (12 points)

Consider the domain of people, with the following predicates:

Oxy means that x is a parent of y , Vx means that x is a woman and Lxy means that x loves y . The predicate $=$ may also be used.

Also, j , a and m are constants that stand for Jan, Ans, and Marie.

Now write each of the following sentences as a formula of predicate logic:

- Ans and Marie are women, but Jan is not.
- Jan has precisely one sister.
- Everyone who has precisely one sister, loves that sister.

6. Semantic tableaux predicate logic. (12 points)

- Explain the $\forall_L$ reduction rule.
- Prove by means of a semantic tableau that
 $\exists x Ax, \forall x \forall y (\neg Ax \vee Cy) \models \forall x Cx$
- The following consequence is *not* valid: $\exists x Ax, \exists x \forall y (\neg Ax \vee Cy) \not\models \forall x Cx$.
Construct by means of a semantic tableau a counterexample for this consequence.

7. (12 points)

- Explain the $\forall I$ rule. Why is the condition essential?
Prove with the aid of natural deduction:
 $\forall x \forall y (Ax \vee By), \forall x (Ax \rightarrow Cx), \forall x (Bx \rightarrow Cx) \vdash \forall x Cx$.
- $\forall x \forall y (Ax \rightarrow Bxy), \exists x Ax \vdash \exists x \forall y Bxy$.

8. (10 points)

Consider the collection of those formulas from the propositional logic which contain only the connective $\wedge$ (that is: $\neg$, $\vee$, $\rightarrow$ and $\leftrightarrow$ do not occur). Let p be a given proposition letter.

Prove with the aid of formula induction that for all these formulas φ holds:
if p occurs in φ then it holds that $\varphi \vdash p$.

9. (10 points)

- Given the model $M = (\mathbf{D}, I)$ consisting of the structure $\mathbf{D} = \langle \mathbf{R}, <, 0, 1, *, + \rangle$ and the interpretation function I , so that $I(R) = <$, $I(a) = 0$, $I(b) = 1$, $I(f) = +$ and $I(g) = *$.
 - Show that $M \models \exists x \forall y R(x, f(g(y, y), b))$.
 - Given the formula $\varphi = \forall y R(x, f(g(y, y), b))$. For which look-up table(s) is this formula true in M ?
- The formula $(\exists x \varphi \wedge \exists x \psi) \rightarrow \exists x (\varphi \wedge \psi)$ is not universally valid.
Give formulas φ , ψ , a model and (if necessary) a look-up table for which $V_{M,b}((\exists x \varphi \wedge \exists x \psi) \rightarrow \exists x (\varphi \wedge \psi)) = 0$.