Universiteit Maastricht
Faculty of General Sciences
Knowledge Engineering

Exam

Logic

Friday 3 June 2005

This exam is NOT an open-book exam.
You will get an auxiliary sheet with the most important formulas.

Clarify all your answers sufficiently!
Indicate in semantic tableaux exactly which reduction rules you are using.
In the case of derivations, indicate exactly from which formula('s) each formula is derived and if the conditions are met (if applicable).

This exam contains 9 assignments. For each assignment will be indicated the maximum amount of points that you can obtain for a right answer. The exam mark is the number of points for all assignments together, divided by 10.

1. (10 points)
   About the weather in Amsterdam we know the following:
   i. If it is raining, then it is cloudy.
   ii. If the sun shines, then it is not cloudy.
   iii. It is raining or it is cloudy.

   Does the sun shine in Amsterdam?
   Determine the answer to these questions (yes or no) with the aid of a truth table (model elimination).

2. Semantic tableaux propositional logic. (12 points)
   a. Explain the $\vee_R$ reduction rule.
   b. Given is the consequence $p \wedge (q \rightarrow r) \;\;/\;\; \neg p \;\vee\; (q \wedge r)$. Investigate the validity of this consequence with the aid of a semantic tableau. If this consequence is not valid, give all the counterexamples.
   c. Investigate by means of a semantic tableau whether the formula $((p \vee q) \wedge (p \rightarrow r)) \rightarrow ((q \wedge \neg r) \vee r)$ is a tautology (if is not a tautology, then give a counterexample).

3. Natural deduction (propositional logic) (12 points).
   a. Explain how to apply the $\vee$E rule.
   Prove by means of natural deduction:
   b. $p \rightarrow q,\; q \rightarrow s \vdash (p \wedge r) \rightarrow s$
   c. $p \vee q,\; p \rightarrow r,\; \neg q \vdash r$

4. (10 points)
   Translate the following formula into an equivalent formula in prenex form:
   $(((\exists x\, \forall y\, A(x,y)) \rightarrow (\exists y\, B(x,y))) \wedge ((\exists y\, B(x,y)) \rightarrow (\forall x\, \exists y\, A(x,y))))$

5. (12 points)
   Consider the domain of animals on a farm, with the following predicates:
   Cx means that x is a cow, Hx means that x is a horse, Gx means that x is a goat, and Bx means that x is black. The predicate = may also be used.
   Now write each of the following sentences as a formula of predicate logic (don't use the quantor $\exists!$; the quantors $\exists$ and $\forall$ can do the job):

   a. There is a cow.
   b. There is precisely one horse.
   c. If there is precisely one goat, then that goat is black.

6. Semantic tableaux predicate logic. (12 points)
   a. Explain the $\exists_R$ reduction rule.
   b. Prove by means of a semantic tableau that $\exists x\, Ax,\; \forall x\, (\neg Ax \vee Cx) \models \exists x\, Cx$
   c. The following consequence is *not* valid: $\exists x\, Ax,\; \forall x \forall y\, (Ax \rightarrow By) \;\;/\; \forall x\, Bx$.
   Construct by means of a semantic tableau a counterexample for this consequence.
   Sorry, it is valid!

7. (12 points)
   a. Explain how to use the $\exists$E rule.
   Prove with the aid of natural deduction:
   b. $\forall x\, Ax,\; \forall x\, (Ax \rightarrow Bx) \vdash \forall x\, (Ax \wedge Bx)$.
   c. $\forall x\, (Bx \rightarrow Cx),\; \exists x\, Bx \vdash \exists x\, Cx$.

8. (10 points)
   Consider the collection **C** of those formulas from the propositional logic which contain only the connective $\wedge$ and $\vee$ (that is: $\neg$, $\rightarrow$ and $\leftrightarrow$ do not occur).
   Consider a valuation function V, such that $V(p) = 1$ for all proposition letters p.
   Prove with the aid of formula induction that for all formulas φ from **C** it holds that $V(\varphi) = 1$.

9. (10 points)
   a. Given the model $M = (\boldsymbol{D}, I)$ consisting of the structure $\boldsymbol{D} = \langle \mathbf{R}, <, 2, 5, *, + \rangle$ and the interpretation function I, so that $I(R) = <$, $I(a) = 2$, $I(c) = 5$, $I(f) = +$ and $I(g) = *$.
   i. Show that $M \models \exists x\, \forall y\, R(\, x, f(g(y,y), f(g(a,y),c))\,)$.
   ii Given the formula $\varphi = \forall y\, R(\, x, f(g(y,y), f(g(a,y),c))\,)$. For which look-up table(s) is this formula true in M?
   b. The formula $((\exists x\, (\varphi \rightarrow \psi)) \rightarrow ((\exists x\, \varphi) \rightarrow \psi)\,)$ is not universally valid.
   Give formulas φ, ψ, a model and (if necessary) a look-up table for which $V_{M,b}(\, ((\exists x\, (\varphi \rightarrow \psi)) \rightarrow ((\exists x\, \varphi) \rightarrow \psi)\,)\,) = 0$.