Universiteit Maastricht
Faculty of Humanities and Sciences
Knowledge Engineering

Exam

Logic

Friday 29 June 2007, 9.00-12.00

This exam is NOT an open-book exam.
You will get an auxiliary sheet with the most important formulas.

Clarify all your answers sufficiently!
Indicate in semantic tableaux exactly which reduction rules you are using.
In the case of derivations, indicate exactly from which formula(s) each formula is derived and if the conditions are met (if applicable).

This exam contains 9 assignments. For each assignment the maximum score that you can obtain for a correct answer is indicated. The final score for the exam is the sum of the scores for the individual assignments, divided by 10.

1. (10 points).
   When Eric eats a hamburger, his choice of condiments is determined by the following facts:
   i. Eric takes ketchup or mayonnaise on his hamburger.
   ii. If Eric takes onions or ketchup, he will also take mayonnaise.
   iii. If Eric takes mayonnaise and onions, he will not take ketchup.
   iv. If Eric does not take onions, he will not take ketchup.

   Does Eric take ketchup? Does he take mayonnaise?
   Determine the answer with the aid of a truth table (model elimination).

2. Semantic tableaux propositional logic (12 points).
   a. Explain the $\wedge_R$ reduction rule.
   b. Given is the consequence $p \to (q \wedge r), (p \vee q) \to \neg r \ / \ r \to \neg p$.
      Investigate the validity of this consequence with the aid of a semantic tableau. If this consequence is not valid, give all the counterexamples.
   c. Investigate by means of a semantic tableau whether the formula $(\neg p \wedge q) \leftrightarrow (p \to q)$ is a tautology. If it is not a tautology, then give all counterexamples.

3. Natural deduction propositional logic (12 points).
   Prove by means of natural deduction:
   a. $(p \vee q) \to r, s \to q, (r \wedge s) \to p \vdash (p \wedge s) \to (q \wedge r)$
   b. $p \to r, q \to (p \vee r) \vdash (p \vee q) \to r$
   c. $(p \to q) \to r, p \to r \vdash r$

4. (10 points).
   Translate the following formula into an equivalent formula in prenex form:
   $\forall x((\forall x \mathrm{P}xy \to (\exists x \mathrm{Q}xy \to \neg \forall y \mathrm{R}xy)) \to (\neg \exists x \mathrm{S}x \wedge \mathrm{T}x))$

5. (12 points).
   Consider the domains of cooks and dishes. Pxy means that dish x is prepared by cook y. Lxy means that cook y likes dish x. The predicate = may also be used.
   Now write each of the following sentences as a formula of predicate logic:
   a. Every cook likes at least one dish.
   b. No cook likes the dishes he prepared himself.
   c. There is a cook who likes all dishes that are prepared by cooks who like all the dishes that he has prepared.

6. Semantic tableaux predicate logic (12 points).
   a. Explain the $\forall_R$ reduction rule.
   Investigate the validity of the following consequences by means of a semantic tableau. If a consequence is not valid, give at least one counterexample.
   b. $\forall x((\mathrm{A}x \wedge \mathrm{B}x) \rightarrow \mathrm{C}x), \neg\exists x\mathrm{C}x \,/\, \forall x(\neg\mathrm{A}x \vee \neg\mathrm{B}x)$
   c. $\forall x(\mathrm{A}x \vee \mathrm{B}x), \exists x(\mathrm{B}x \rightarrow \mathrm{C}x), \neg\exists x\mathrm{A}x \,/\, \neg\exists x\mathrm{B}x$

7. Natural deduction predicate logic (12 points).
   a. Explain the $\exists$E rule, and make clear why the condition is essential.
   Prove by means of natural deduction:
   b. $\forall x(\mathrm{A}x \vee \mathrm{B}x), \neg\exists x\mathrm{B}x \vdash \forall x\mathrm{A}x$
   c. $\forall x(\neg(\mathrm{A}x \wedge \mathrm{B}x)), \exists x\mathrm{A}x \vdash \exists x\neg\mathrm{B}x$

8. (10 points).
   Consider two new logical symbols of the propositional logic: $\uparrow$ and $\oplus$. The symbol $\uparrow$ is the so-called NAND symbol. $(p \uparrow q)$ is false if and only if p and q are both true. The symbol $\oplus$ is the so-called XOR symbol. $(p \oplus q)$ is false if and only if p and q are equal.
   a. Prove that the set $\{\uparrow\}$ is functionally complete. You may use the fact that all formulas of the propositional logic can be expressed in disjunctive normal form.
   b. Is the set $\{\oplus\}$ functionally complete? Provide a solid argument for your answer.

9. (10 points).
   Given are models $M_1 = (\boldsymbol{D}_1, I)$, $M_2 = (\boldsymbol{D}_2, I)$, $M_3 = (\boldsymbol{D}_3, I)$, and $M_4 = (\boldsymbol{D}_4, I)$.
   Structure $\boldsymbol{D}_1 = \langle\mathbb{N}, <\rangle$, structure $\boldsymbol{D}_2 = \langle\mathbb{Z}, <\rangle$, structure $\boldsymbol{D}_3 = \langle\mathbb{Q}, <\rangle$, and structure $\boldsymbol{D}_4 = \langle\mathbb{R}, <\rangle$. The interpretation function is defined as $I(R)=<$.
   a. For which lookup-table(s) is the formula $\neg\exists x\mathrm{R}xy$ true in M_1?
   b. Find a formula φ such that M_1, M_2, M_3, and M_4 are all models of φ.
   c. Find a formula ψ such that M_1 and M_2 are not models of ψ, but M_3 and M_4 are models of ψ.
   d. Is it possible to give a formula of which M_3 is a not model, but M_4 is a model? If so, give such a formula. If not, provide a solid argument for your answer.