Maastricht University
Faculty of Humanities and Sciences
Knowledge Engineering

Logic 2007/2008

Exam

Wednesday, June 4, 2008, 9.00–12.00h

This exam is NOT an open-book exam. An auxiliary sheet with the most important formulas is added to the exam.

Clarify all your answers sufficiently! Indicate in semantic tableaux exactly which reduction rules you are using. In the case of derivations, indicate exactly from which formula(s) each formula is derived and if the conditions are met (if applicable).

This exam contains nine assignments. For each assignment the maximum score that you can obtain for a correct answer is indicated. The final score for the exam is the sum of the scores for the individual assignments, divided by 10.

1. (10 points)
   John, Liz, and Anne have a day off and consider going to the beach.
   i. Anne will not go to the beach alone.
   ii. If both Anne and John go to the beach, Liz will not go.
   iii. If Liz goes to the beach, Anne will go too.
   iv. John will go where Liz goes.

   Will John go to the beach? Will Liz? Will Anne?
   Determine the answers with the aid of a truth table (model elimination).

2. (12 points)
   Determine the answers to the following questions using semantic tableaux:
   (a) Consider the consequence $(p \vee q) \leftrightarrow (\neg p \wedge r) \;/\; \neg q \rightarrow r$.
   Is this consequence valid? If the consequence is not valid, give all the counterexamples.
   (b) Consider the set of formulas $\{p \wedge \neg q, (p \wedge r) \rightarrow q, r\}$.
   Is this set consistent? If the set is consistent, then give a valuation which is a model of the set.

3. (12 points)
   Prove by means of natural deduction:
   (a) $(p \vee q) \rightarrow r, (p \wedge r) \rightarrow q, p \vdash (p \wedge q) \wedge r$
   (b) $p \rightarrow \neg q, (q \wedge r) \rightarrow p \vdash r \rightarrow \neg q$
   (c) $(p \vee (q \rightarrow r)) \vee r, (p \rightarrow r) \wedge q \vdash r$

4. (10 points)
   Transform the following formula to an equivalent formula in prenex form:
   $\neg((\forall x Pxy \rightarrow \forall x(\exists y Qxy \rightarrow \forall x Rxy)) \wedge \neg\exists x Sx)$

5. (12 points) Consider the domain of pirates and ninjas. Px means that x is a pirate. Nx means that x is a ninja. Hxy means that x hates y. The predicate $=$ may also be used. Now write each of the following sentences as a formula of the predicate logic:

   (a) Every pirate hates at least one ninja.

   (b) There is a pirate who does not hate all ninjas.

   (c) There is a pirate who hates all pirates except for himself.

6. (12 points)
   Investigate the validity of the following consequences by means of semantic tableaux. If a consequence is not valid, give at least one counterexample.

   (a) $\exists x \forall y (Ax \to Bxy), \forall x Ax \;/\; \forall x Bxx$

   (b) $\forall x Bxx \;/\; \exists x \forall y (Bxy \vee Byx)$

7. (12 points)
   Prove by means of natural deduction:

   (a) $\exists x \forall y (Ax \to Bxy), \forall x Ax \vdash \exists x Bxx$

   (b) $\forall x (Ax \vee Bx), \neg \exists x Ax \vdash \forall x Bx$

8. (10 points)
   Given are the models $M_1 = (\mathbf{D}, I_1)$ and $M_2 = (\mathbf{D}, I_2)$.

   Structure $\mathbf{D} = \langle \mathbb{R}, \leq, 1, 0, \cdot, + \rangle$.

   Interpretation function I_1 is defined as:
   $I_1(R) = \leq, I_1(a) = 1, I_1(b) = 0, I_1(f) = \cdot$, and $I_1(g) = +$.

   Consider formula $\varphi = \exists y R(g(f(x, g(x, b)), f(y, y)), f(a, a))$.

   (a) For which look-up tables b does it hold that $M_1, b \models \varphi$?

   (b) Define I_2 in such a way that $M_2, b \models \varphi$ holds for any look-up table b.

9. (10 points)
   Let φ^d be the result of prefixing every proposition letter in φ with $\neg$, and replacing every $\vee$ in φ with $\wedge$. For example, if φ is $(\neg p \vee q)$, then φ^d is $(\neg\neg p \wedge \neg q)$. Moreover, if φ is $(\neg p \wedge q)$, then φ^d is also $(\neg\neg p \wedge \neg q)$.

   Using formula induction, prove that, for any set of formulas Σ and any formula φ of the propositional logic, it holds that: $\Sigma \models \varphi \Rightarrow \Sigma \models \neg\varphi^d$.

   The second step of the proof you only need to give for the logical operators $\neg$, $\wedge$, and $\vee$. The proof for $\to$ and $\leftrightarrow$ follows from the fact that you can write any formula of the propositional logic in disjunctive normal form.

Good luck!