

Spline Interpolation

$$\begin{cases} \rightarrow S(x_j) = a_j \\ \rightarrow S'(x_j) = b_j \\ \rightarrow S''(x_j) = 2c_j \end{cases} \quad \left| \quad \begin{aligned} S'''(x_j^-) &= 6d_j^- \\ S'''(x_j^+) &= 6d_j^+ \end{aligned} \right.$$

Compute the spline interpolating $y_i = f(x_i)$ for

i	0	1	2	3
x_i	1	2	3	5
$a_i = y_i$	2	4	7	8

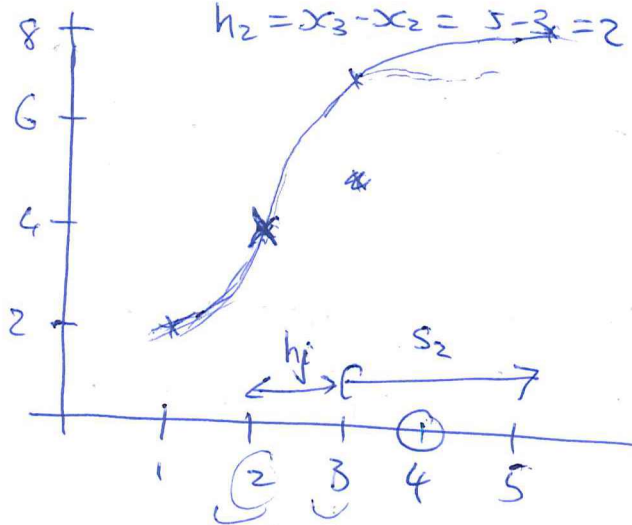
End
~~Boundary~~ conditions

$$S'(x_0) = 1 = b_0$$

$$S''(x_3) = 0 = 2c_3$$

$$h_0 = x_1 - x_0 = 2 - 1 = 1 \quad h_1 = x_2 - x_1 = 3 - 2 = 1$$

$$h_2 = x_3 - x_2 = 5 - 3 = 2$$



On $[x_i, x_{i+1}] \quad i=0, 1, 2$

$$S_j(x) = a_j + (x - x_j)b_j + \frac{(x - x_j)^2}{h_j}c_j + \frac{(x - x_j)^3}{h_j^2}d_j$$

$$h_j = x_{j+1} - x_j$$

$$j=0 \quad 2h_0c_0 + h_0c_1 = 3\left(\frac{a_1 - a_0}{h_0} - b_0\right)$$

$$j=1 \quad h_0c_0 + 2(h_0 + h_1)c_1 + h_1c_2 = \frac{3}{h_1}(a_2 - a_1) + \frac{3}{h_0}(a_1 - a_0)$$

$$j=2 \quad h_1c_1 + 2(h_1 + h_2)c_2 + h_2c_3 = \frac{3}{h_2}(a_3 - a_2) + \frac{3}{h_1}(a_2 - a_1)$$

$$j=3 \quad c_3 = \frac{0}{2} = 0$$

$$j=0: \quad 2c_0 + c_1 = 3 \times \left(\frac{4-2}{1} - 1 \right) = 3 \times (2-1) = 3$$

$$j=1: \quad c_0 + 4c_1 + c_2 = \frac{3}{1}(7-4) + \frac{3}{1}(4-2) = 3 \times 3 + 3 \times 2 = 15$$

$$j=2: \quad c_1 + 6c_2 + 2c_3 = \frac{3}{2}(8-7) + \frac{3}{1}(7-4) = 10.5$$

$$c_3 = 0$$

$$2c_0 + c_1 = 3$$

$$c_0 + 4c_1 + c_2 = 15$$

$$c_1 + 6c_2 = 10.5$$

$$\begin{pmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 6 \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 15 \\ 10.5 \end{pmatrix}$$

$$c_0 = -0.2625$$

$$= \frac{-21}{80}$$

$$c_1 = 3.525$$

$$= \frac{141}{40}$$

$$c_2 = 1.1625$$

$$= \frac{93}{80}$$

$$c_3 = 0$$

To approximate y when $x=4$, need piece

$$S_2(x) = a_2 + (x-x_2)b_2 + (x-x_2)^2 c_2 + (x-x_2)^3 d_2$$

$\parallel$ $\parallel$
 7 1.1625

$$b_2 = \frac{(a_3 - a_2)}{h_2} - \frac{h_2}{3} (2c_2 + c_3)$$

$$= \frac{8-7}{2} - \frac{2}{3} (2 \times 1.1625 + 0)$$

$$= -1.05$$

$$d_3 = \frac{c_3 - c_2}{3h_2}$$

$$= \frac{0 - 1.1625}{3 \times 2} = -0.19375$$

$$\approx -0.1938$$

$$S_2(x) = 7 + (-1.05)(x-3) + 1.1625(x-3)^2 + (-0.19375)(x-3)^3$$

$$S_2(4) = 7 + (-1.05) \times 1 + 1.1625 \times 1^2 + (-0.19375) \times 1^3$$

$$= 6.91875 = 6.9 \text{ (1dp)}$$

$$S_2(5) = 8 = y_3 \quad \checkmark$$