DKE Numerical Mathematics
Sample Exam 2012

Part I

Time permitted: $1\frac{1}{2}$ hours.
Additional material: You may only use the formula sheet provided.
Electronic calculators: Policy to be determined.

1. **[8 points]** Use two iterations of Newton's algorithm to estimate the root of $x^3 - x - 1 = 0$, starting your iteration at $x = 1$.

2. **[8 points]** Compute polynomial interpolating the following data:

$$\begin{array}{c|c|c|c|c} x & 2 & 3 & 1 & 4 \\ \hline y & 7 & 4 & 8 & 5 \end{array}.$$

   Write your answer in nested form. Approximate the value of the data when $x = 2.5$.

3. **[8 points]** Compute the best linear approximation $l(x)$ in the least-squares sense to the following data:

$$\begin{array}{c|c|c|c|c|c} x & 1 & 2 & 4 & 5 & 6 \\ \hline y & 2 & 5 & 4 & 7 & 9 \end{array}.$$

   Use $l(x)$to estimate the value of y when $x = 3$.

4. **[8 points]** Estimate $\int_0^2 f(x)\,dx$ by applying Simpson's rule to the following data:

$$\begin{array}{c|c|c|c|c|c} x & 0.0 & 0.5 & 1.0 & 1.5 & 2.0 \\ \hline f(x) & 1.14 & 1.23 & 1.53 & 1.92 & 2.34 \end{array}.$$

   Estimate the error of your answer assuming $\max_{\xi\in[0,2]} |f^{(4)}(\xi)| \leq 41$.

5. **[8 points]** Use two steps of Heun's method to estimate the solution of the differential equation $\dot{x} = 2x - t^2$ with $x(1) = 1$ at $t = 2$.

Part II

Time permitted: $1\frac{1}{2}$ hours.
Additional material: You may only use the formula sheet provided.
Computer use: You may *only* use the DKE computers, and *only* use Matlab.

6. **[10 Points]** Using built-in Matlab commands, or otherwise, compute the cubic spline interpolating the following data:

x	0	1	2	3	4	5
y	6.4	5.1	3.2	2.0	1.3	0.9

   Use the boundary conditions $s'(0) = 0$ and $s'(5) = 0.3$. Write down the formula for $s(x)$ for $3 \leq x \leq 4$. Estimate the value of y when $x = 3.2$.

7. **[10 Points]** Compute the Romburg estimate $R_{4,4}$ to the definite integral

$$\int_1^4 \sin(x^2)\, dx.$$

   Write out the computed values of $R_{i,j}$ to 6 decimal places.

8. **[10 Points]** Compute the solution of the ordinary differential equation

$$\dot{x}(t) = x - t^2; \quad x(0) = 1$$

   over the interval $[0, 2]$ using the two-step Adams-Bashforth method as a predicator, and the two-stage Adams-Moulton method as a corrector. Use a step size $h = 0.4$, and write down all your computed predicted and corrected values.

9. **[10 Points]** Consider the ordinary differential equation

$$\dot{x} = -400x + 12t; \quad x(0) = 1.$$

   Use the Matlab solvers `ode23` and `ode23s` to compute $x(6)$. Compare the properties of the two methods, both in terms of the number of steps taken, and the accuracy obtained. Which method is better? Explain your answer.