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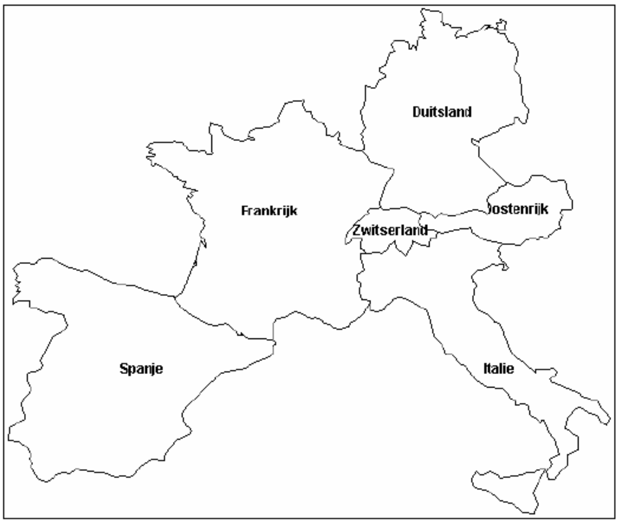
TA in Reasoning Techniques 2023

EXAM1 Task 1.

The map-3-colouring problem is a variant of the graph-3-colouring problem. The task is to colour all countries on a map in such a way that neighbouring countries have different colours. Use the following map:



a. Define the problem to colour this map with only 3 colours, i.e., red, blue and yellow, as a CSP problem <*V, D, R*>. State clearly the contents of the sets *V*, *D* and *R*.

Note: the lecture slides are about <V, D, C>. I suspect this exam to have been translated from Dutch, with the R standing for Restricties, Constraints.

Variables: V={S, F, D, Z, O, I}

Domain : D={R, B, Y}

Constraints: R={(Vi,Vj),{(R,B),(R,G),(B,R),(B,G),(G,R),(G,B)}} for every edge (i,j)

In words: adjacent regions must have different colors

b. Simulate the process of the standard backtrack procedure using the following (vertical) ordering for the countries: {Spanje, Oostenrijk, Italië, Frankrijk, Duitsland, Zwitserland} and (horizontal) ordering {red, blue, yellow} for the colours of each country. Give explicitly and in the right order which country each time gets which colour and state when an assignment fails or has success. How many assignments do you need to find a solution and how often did you have to backtrack?

Read row by row left to right

S: R O: I: F: D: Z:	S: R O: R I: F: D: Z:	S: R O: R I: B F: D: Z:	S: R O: R I: B F: D: Z:	S: R O: R I: B F: Y D: Z:	S: R O: R I: B F: Y D: B Z:	S: R O: R I: B F: Y D: B Z: **X**	S: R O: R I: B F: Y D: **X** Z:	S: R O: R I: B F: **X** D: Z:	S: R O: R I: Y F: D: Z:	S: R O: R I: Y F: B D: Z:
S: R O: R I: Y F: B D: Y Z:	S: R O: R I: Y F: B D: Y Z: **X**	S: R O: R I: Y F: B D: **X** Z:	S: R O: R I: Y F: **X** D: Z:	S: R O: R I: **X** F: D: Z:	S: R O: B I: F: D: Z:	S: R O: B I: R F: D: Z:	S: R O: B I: R F: B D: Z:	S: R O: B I: R F: B D: R Z:	S: R O: B I: R F: B D: R Z: Y	

c. Do the same for the application of the forward-check method, using the same horizontal and vertical orderings as in part b. State at each moment clearly the contents of the domains of the variables after every assignment.

S: **R** O: RBY I: RBY F: BY D: RBY Z: RBY	S: **R** O: **R** I: BY F: BY D: BY Z: BY	S: **R** O: **R** I: **B** F: **Y** D: BY Z:	S: **R** O: **R** I: **Y** F: **B** D: BY Z:	S: **R** O: **B** I: RY F: BY D: RY Z: RY	S: **R** O: **B** I: **R** F: **B** D: **R** Z: **Y**

EXAM1 Task 2.

Given are the following facts:

• if pushable objects are blue, then non-pushable objects are green;

- all objects are either blue or green, but not both;
- if there exists a non-pushable object, then all pushable objects are blue;
- object O1 is pushable;
- object O2 is non-pushable.

a. Bring this information in predicate form.
Blue(x) := Object X is blue
Green(x) := Object X is green
Pushable(x) := Object X is pushable

- if pushable objects are blue then non-pushable ones are green.

$(\forall x : Pushable(x) \Rightarrow Blue(x)) \Rightarrow (\forall y : \neg Pushable(y) \Rightarrow Green(y))$

- all objects are either blue or green but not both.

$(\forall x : Green(x) \lor Blue(x)) \land (\forall x : \neg Blue(x) \lor \neg Green(x))$

- if there is a non-pushable object, then all pushable objects are blue

$(\exists x : \neg Pushable(x)) \Rightarrow (\forall y : Pushable(y) \Rightarrow Blue(y))$

- object O1 is pushable

Pushable(O1)

- object O2 is non-pushable

¬Pushable(O2)

b. Next bring the predicate assertions in *conjunctive normal form* (CNF).
$(\forall x : Pushable(x) \Rightarrow Blue(x)) \Rightarrow (\forall y : \neg Pushable(y) \Rightarrow Green(y))$
$\neg(\forall x : \neg Pushable(x) \lor Blue(x)) \lor (\forall y : Pushable(y) \lor Green(y))$ *implication elimination*
$(\exists x : Pushable(x) \land \neg Blue(x)) \lor Pushable(y) \lor Green(y)$ *quantifier duality, implicit uniform*
$(Pushable(S) \land \neg Blue(S)) \lor Pushable(y) \lor Green(y)$ *existential instantiation*
$(Pushable(S) \lor Pushable(y) \lor Green(y)) \land (\neg Blue(S) \lor Pushable(y) \lor Green(y))$ *distributivity*

$(\forall x : Green(x) \lor Blue(x)) \land (\forall x : \neg Blue(x) \lor \neg Green(x))$
$(Green(x) \lor Blue(x)) \land (\neg Blue(x) \lor \neg Green(x))$ *implicit uniform*

$(\exists x : \neg Pushable(x)) \Rightarrow (\forall y : Pushable(y) \Rightarrow Blue(y))$
$\neg(\exists x : \neg Pushable(x)) \lor (Pushable(y) \Rightarrow Blue(y))$ *implication elimination*
$(\forall x : (Pushable(x))) \lor (\neg Pushable(y) \lor Blue(y))$ *quantifier duality*
$Pushable(x) \lor \neg Pushable(y) \lor Blue(y)$ *implicit uniform*

Pushable(O1)

¬Pushable(O2)

c. Give a resolution proof, ~~using the unit-preference strategy~~, that there exists a green object.
Proof by contradiction, i.e., show "There is not a green object".
$\forall x : \neg Green(x)$
$\neg Green(x)$ *implicit uniform*

$$\frac{\neg \mathbf{Pushable}(\mathbf{O2}),\ \mathbf{Pushable}(\mathbf{x}) \lor \neg \mathbf{Pushable}(\mathbf{y}) \lor \mathbf{Blue}(\mathbf{y})}{\neg \mathbf{Pushable}(\mathbf{y}) \lor \mathbf{Blue}(\mathbf{y})}$$

$$\frac{\neg \mathbf{Pushable}(\mathbf{y}) \lor \mathbf{Blue}(\mathbf{y}),\ (\mathbf{Pushable}(\mathbf{S}) \lor \mathbf{Pushable}(\mathbf{y}) \lor \mathbf{Green}(\mathbf{y})) \land (\neg \mathbf{Blue}(\mathbf{S}) \lor \mathbf{Pushable}(\mathbf{y}) \lor \mathbf{Green}(\mathbf{y}))}{\mathbf{Pushable}(\mathbf{y}) \lor \mathbf{Green}(\mathbf{y})}$$

$$\frac{\neg \mathbf{Pushable}(\mathbf{O2}), \mathbf{Pushable}(\mathbf{y}) \lor \mathbf{Green}(\mathbf{y})}{\mathbf{Green}(\mathbf{O2})}$$

$$\frac{\mathbf{Green}(\mathbf{O2}), \neg \mathbf{Green}(\mathbf{x})}{\boldsymbol{NIL}}$$

EXAM2 Task 4.

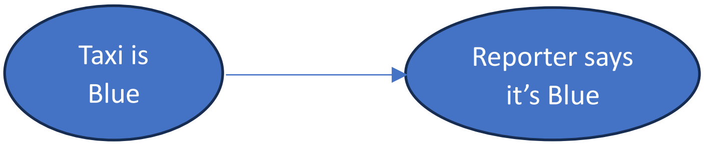
In a galaxy far, far away, 90% of the taxicabs are green and 10% are blue. An accident involving a taxicab occurs; we presume that the accident rate for green taxicabs is the same as that for blue taxicabs.

A court considers the accident, and a newspaper reporter who was at the scene says: “The cab involved was blue.” This reporter is usually reliable; in fact, his statements are correct 80% of the time. That is, if the taxicab involved in the accident was in fact blue (or green), the probability that our witness would say “blue” (or “green”) is 0.8.

(a) Draw a Bayes Belief Network modelling this.

(b) Give relevant conditional probability tables.

P(B)
0.1



B	P(R|B)
T	0.8
F	0.2

(c) What is the probability that the taxicab involved in the accident was blue, given the reporter’s statement?

Bayes’ rule gives us:

$$P(B|R) = \frac{P(R|B)P(B)}{P(R)} = \frac{0.8 \times 0.1}{0.26} = 0.3077$$

The Chain rule gives us:

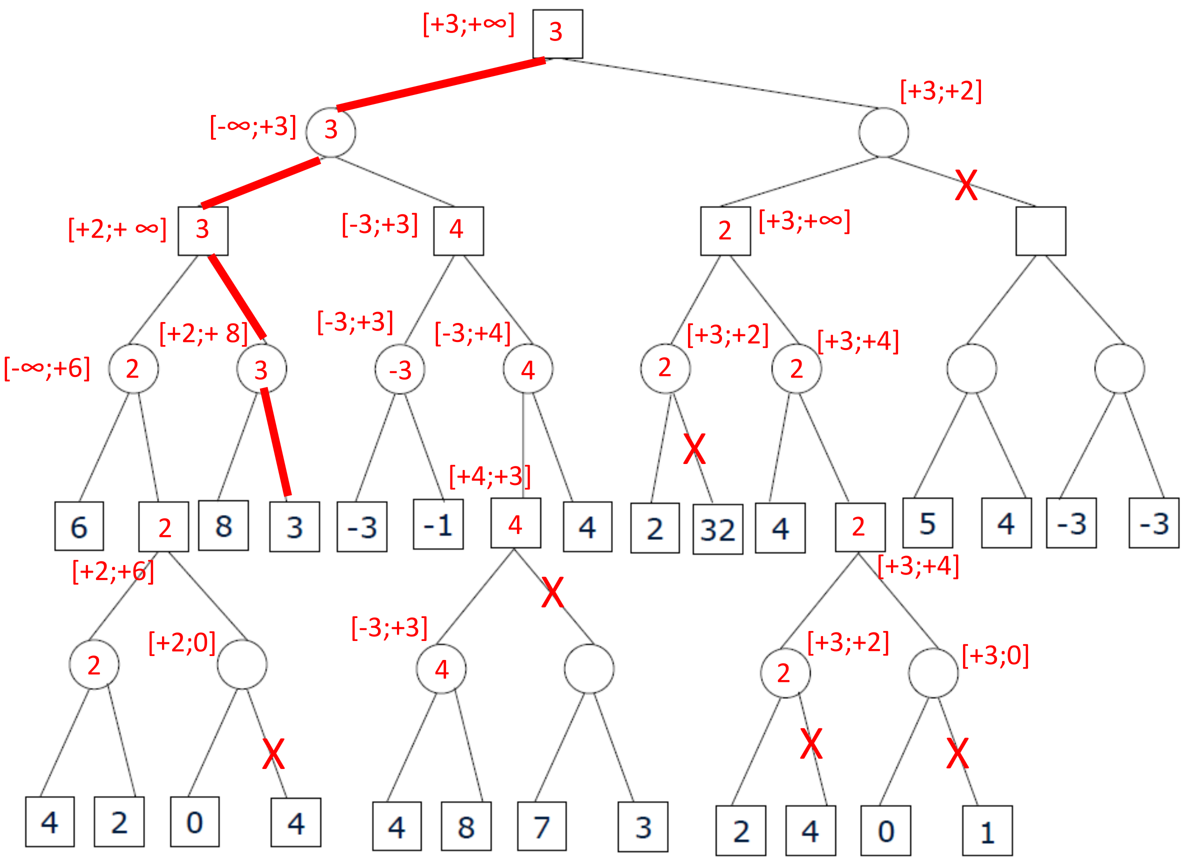
$$P(R) = P(R|B)P(B) + P(R|\neg B)P(\neg B) = 0.8 \times 0.1 + 0.2 \times 0.9 = 0.26$$

EXAM3 Search and Games

Assume that the following tree is traversed from left to right. MAX nodes are denoted by a square, MIN nodes are denoted by a circle. The values in the leaf nodes represent evaluation scores.

a) Apply the αβ algorithm on this game tree above. Denote the branches that are pruned by X in the tree above. Indicate the bounds in the tree as well.

b) Show the optimal line of play for both players in the game tree.



c) Will the same αβ prunings occur in case the evaluation scores are subjected to a monotonic transformation? Explain your answer.

The αβ prunings will remain identical as a monotonic transformation won't change anything to the order of the branches, resulting so in new bounds but with the same nodes as upper/lower bounds.

EXAM3 First-order logic

a) Given is the following text:

Anyone who buys wholesale fish owns either a cat or a market stall. Every dog chases some cat.

Lisa buys wholesale fish. Anyone who owns a cat hates anything that chases any cat. Danny owns a dog. Someone who hates something owned by another person will not date that person. If Lisa does not own a market stall, she will not date Danny.

Bring this information in predicate (first-order logic) form, giving a literal translation of each sentence.

Let:

- B(x):= x buys wholesale fish.
- O(x, y):= x owns y.
- C(x):= x is a cat.
- D(x):= x is a dog.
- M(x):= x is a market stall.
- H(x, y):= x hates y.
- Ch(x, y):= x chases y.
- P(x, y):= x dates y.

1. Anyone who buys wholesale fish owns either a cat or a market stall.
   $\forall x\ (B(x) \Rightarrow \exists y(O(x,C(y)) \vee O(x,M(y))))$

2. Every dog chases some cat.
   $\forall x\ (D(x) \Rightarrow \exists y\ (C(y) \wedge Ch(x, y)))$

3. Lisa buys wholesale fish.
   $B(Lisa)$

4. Anyone who owns a cat hates anything that chases any cat.
   $\forall w,x,y\ ((\exists z\ (O(x, C(z)) \wedge Ch(w, C(y))) \Rightarrow H(x, w))$

5. Danny owns a dog.
   $\exists x\ O(Danny,D(x))$

6. Someone who hates something owned by another person will not date that person.
   $\forall x,y\ ((\exists z\ (O(x, z) \wedge H(y, z)) \Rightarrow \neg P(x, y))$

7. If Lisa does not own a market stall, she will not date Danny.
   $(\neg(\exists z\ O(Lisa, M(z)))\Rightarrow \neg\ P(Lisa, Danny)$

b) Consider the following axioms:

1. $\forall x\ \forall y\ (Child(x) \wedge Dessert(y) \Rightarrow Love(x,y))$
2. $\forall x\ ((\exists y\ (Dessert(y) \wedge Love(x,y))) \Rightarrow \neg OnDiet(x))$
3. $\forall x\ ((\exists y\ (Seeds(y) \wedge Eat(x,y))) \Rightarrow OnDiet(x))$
4. $\forall x\ \forall y\ (Seeds(y) \wedge Buy(x,y) \Rightarrow Plant(x,y) \vee Eat(x,y))$
5. $\exists x\ (Seeds(x) \wedge Buy(Sam,x))$
6. Dessert(Vlaai)

Bring the axioms in conjunctive normal form (CNF) and use CNF resolution on the axioms to prove the following:

Child(Sam) $\Rightarrow \exists x\ (Seeds(x) \wedge Plant(Sam, x))$

1. $\forall x\ \forall y\ (Child(x) \wedge Dessert(y) \Rightarrow Love(x,y))$

$\neg(Child(x) \wedge Dessert(y)) \vee Love(x,y)$ *implication elimination, implicit uniform*

$\boldsymbol{\neg Child(x) \vee \neg Dessert(y) \vee Love(x,y)}$ *De Morgan, associativity of ∨*

2. ∀x ((∃y (Dessert(y) ∧ Love(x,y))) ⇒ ¬OnDiet(x))
¬(∃y (Dessert(y) ∧ Love(x,y))) ∨ ¬OnDiet(x) *implication elimination, implicit uniform*
∀y (¬Dessert(y) ∨ ¬Love(x,y)) ∨ ¬OnDiet(x) *De Morgan, quantifier duality*
¬Dessert(y) ∨ ¬Love(x,y) ∨ ¬OnDiet(x) *implicit uniform, associativity of ∨*

3. ∀x ((∃y (Seeds(y) ∧ Eat(x,y))) ⇒ OnDiet(x))
¬(∃y (Seeds(y) ∧ Eat(x,y))) ∨ OnDiet(x) *implication elimination, implicit uniform*
∀y (¬Seeds(y) ∨ ¬Eat(x,y)) ∨ OnDiet(x) *quantifier duality, De Morgan*
¬Seeds(y) ∨ ¬Eat(x,y) ∨ OnDiet(x) *implicit uniform, associativity of ∨*

4. ∀x ∀y (Seeds(y) ∧ Buy(x,y) ⇒ Plant(x,y) ∨ Eat(x,y))
¬(Seeds(y) ∧ Buy(x,y)) ∨ (Plant(x,y) ∨ Eat(x,y)) *implication elimination, implicit uniform*
¬Seeds(y) ∨ ¬Buy(x,y) ∨ Plant(x,y) ∨ Eat(x,y) *De Morgan, quantifier duality, associativity of ∨*

5. ∃x (Seeds(x) ∧ Buy(Sam,x))
Seeds(A) ∧ Buy(Sam,A) *existential instantiation*

6. **Dessert(Vlaai)**

Proof by contradiction:
¬(Child(Sam) ⇒ ∃x (Seeds(x) ∧ Plant(Sam, x)))
¬(¬Child(Sam) ∨ ∃x (Seeds(x) ∧ Plant(Sam, x))) *implication elimination*
Child(Sam) ∧ (¬Seeds(x) ∨ ¬Plant(Sam, x)) *De Morgan, quantifier duality*

$$\frac{\mathbf{Dessert(Vlaai)},\ \ \neg\mathbf{Child(x)}\ \vee\ \neg\mathbf{Dessert(y)}\ \vee\ \mathbf{Love(x,y)}}{\neg\mathbf{Child(x)}\ \vee\ \mathbf{Love(x,Vlaai)}}$$

$$\frac{\mathbf{Child(Sam)},\ \ \neg\mathbf{Child(x)}\ \vee\ \mathbf{Love(x,Vlaai)}}{\mathbf{Love(Sam,Vlaai)}}$$

$$\frac{\mathbf{Dessert(Vlaai)},\mathbf{Love(Sam,Vlaai)},\ \ \neg\mathbf{Dessert(y)}\ \vee\ \neg\mathbf{Love(x,y)}\ \vee\ \neg\mathbf{OnDiet(x)}}{\neg\mathbf{OnDiet(Sam)}}$$

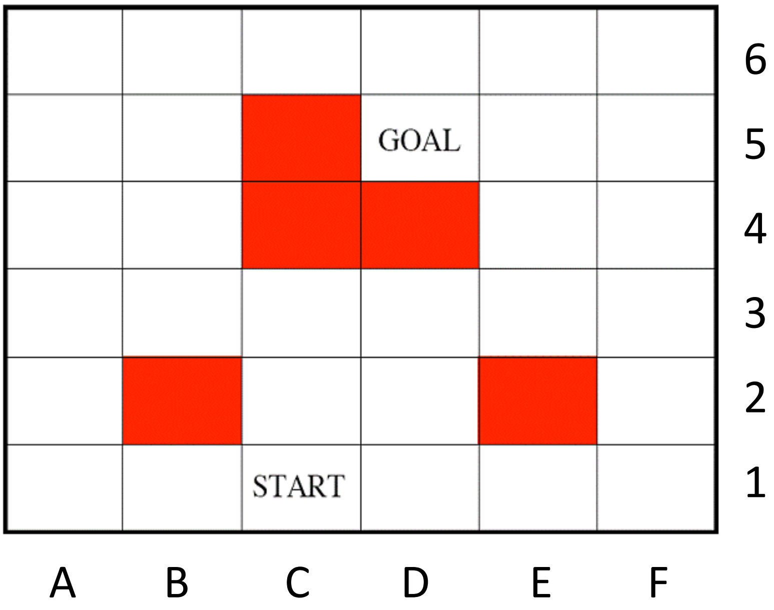
$$\frac{\neg\mathbf{Seeds(y)}\ \vee\ \neg\mathbf{Eat(x,y)}\ \vee\ \mathbf{OnDiet(x)},\ \ \neg\mathbf{OnDiet(Sam)}}{\neg\mathbf{Seeds(y)}\ \vee\ \neg\mathbf{Eat(x,y)}}$$

$$\frac{\mathbf{Seeds(A)}\wedge\ \mathbf{Buy(Sam,A)},\ \ \neg\mathbf{Seeds(y)}\ \vee\ \neg\mathbf{Eat(x,y)}}{\neg\mathbf{Eat(x,A)}}$$

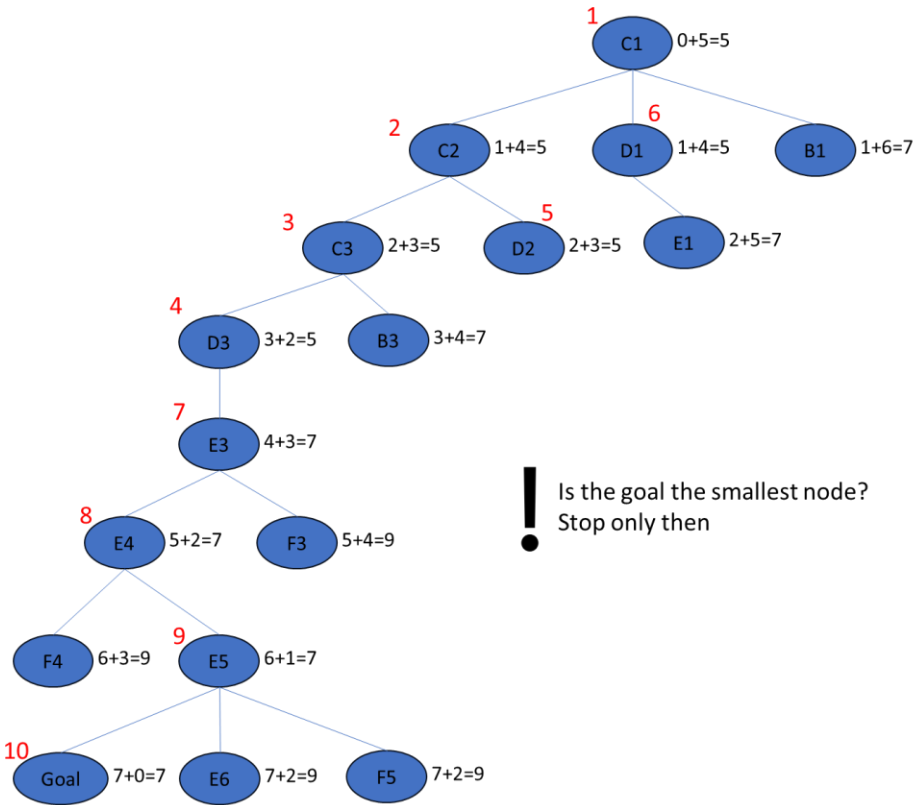
$$\frac{\mathbf{Seeds(A)}\wedge\ \mathbf{Buy(Sam,A)},\neg\mathbf{Eat(x,A)},\neg\mathbf{Seeds(y)}\ \vee\ \neg\mathbf{Buy(x,y)}\ \vee\ \mathbf{Plant(x,y)}\ \vee\ \mathbf{Eat(x,y)}}{\mathbf{Plant(x,A)}}$$

$$\frac{\mathbf{Seeds(A)},\mathbf{Plant(x,A)},\ \ \mathbf{Child(Sam)}\ \wedge\ (\neg\mathbf{Seeds(x)}\ \vee\ \neg\mathbf{Plant(Sam,x)})}{\boldsymbol{NIL}}$$

A*



Find a minimal cost path from the start tile to the goal tile. You can move one square horizontally or vertically, each with a cost of one.



EXAM2 Task 3.
Suppose you want a planning system to figure out how to move cargo between airports. Let's say that you are considering the airports ABQ, SFO, and BOS. You have three cargo objects (C1, C2, and C3) and two airplanes (P1 and P2), all initially located at ABQ. Your goal is for all of the cargo to end up in SFO and both of the planes to end up in BOS. Suppose every airport has exactly one loading site. All loading sites are free initially.
We use the following predicates with obvious meanings:

- `At(X,Y)`: object X is at location Y
- `In(X,Y)`: cargo X is in airplane Y
- `LoadInUse(X)`: the loading site at airport X is in use

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There are 3 possible actions:

- `Load(X,Y,Z)`: load cargo X into plane Y at airport Z;
- `Unload(X,Y,Z)`: unload cargo X from plane Y at airport Z;
- `Fly(X,Y,Z)`: fly airplane X from airport Y to airport Z.
- `Reserve(X,Y)`: reserve the loading site for airplane X at airport Y.
- `Release(X,Y)`: release the loading site for airplane X at airport Y.

Note that before loading or unloading a plane, the loading site should be reserved for that plane and that after loading or unloading a plane, the loading site should be released for that plane before it can take off.
We want to formulate this problem in such a way that a STRIPS-like problem-solving agent can make a plan to reach the goal.
(a) Give a description of the starting situation and the goal situation.
Starting situation: At(C1, ABQ) ∧ At(C2, ABQ) ∧ At(C3, ABQ) ∧ At(P1, ABQ) ∧ At(P2, ABQ) ∧¬LoadInUse(ABQ) ∧ ¬LoadInUse(SFO) ∧ ¬LoadInUse(BOS)
Goal situation: At(C1, SFO) ∧ At(C2, SFO) ∧ At(C3, SFO) ∧ At(P1, BOS) ∧ At(P2, BOS)

(b) Describe the actions in STRIPS format (preconditions, effects).
Action: Load(X, Y, Z)
Preconditions: At(X, Z), At(Y, Z), LoadInUse(Z)
Effects: In(X, Y), ¬At(Y, Z)

Action: Unload(X, Y, Z)
Preconditions: In(X, Y), LoadInUse(Z)
Effects: ¬In(X, Y), At(Y, Z)

Action: Fly(X, Y, Z)
Preconditions: At(X, Y), ¬LoadInUse(Y)
Effects: At(X, Z), ¬At(X, Y)

Action: Reserve(X, Y)
Preconditions: ¬LoadInUse(Y), At(X, Y)
Effects: LoadInUse(Y)

Action: Release(X, Y)
Preconditions: At(X, Y) (might be implied), LoadInUse(Y)
Effects: ¬LoadInUse(Y)

(c) Design a partial-order plan (POP) to reach the goal situation from the start situation.
Take into account *threats* of causal links (*clobbering*) and indicate in the POP how you have solved such threats.
Disclaimer: I am honestly not confident in my ability to give you a fully correct threat resolution of this POP, so I will only give you my insights for this exercise:
Threats are coming from the sequence Plane arrive→Loading Area reserved→Load/Unload→Loading Area released→Plane leaves and from the fact that the cargo must be delivered in SFO before the planes go to BOS.
They can be solved by promoting events in this order (does not mean they all necessarily have to be promoted).
Remember to always connect the predicates, from their origin to the action they precondition. Dangling predicates are an error.