Exam Mathematical Modeling 02/04/2015

- This exam is **Closed book**. You are allowed to use a non-programmable, non-graphical (hence a simple, non cheating) calculator.
- Make sure that your cell phone, tablet, laptop, etc. is on the other side of the room and turned off
- for this exam you can get 100 points plus up to 5 bonus points from the assignment during the practicals. Your grade will be the number of points earned divided by 10
- Good luck!

1. (5 points) Specify a dynamical model, i.e. give the equations, for a situation where you have a single prey population (rabbits) and two predator populations. One of the predators (martens) eat rabbits, but are harmless to the other predator (wolfs). The wolfs though, eat the martens alive (quite literally so). Hence the martens take both the role of prey and predators. Obviously, we are expecting equations/a model that fits in the framework as discussed in the lectures.

2. Consider a linear input-output system in discrete-time given by a second order difference equation of the form:

$$y[k] + \frac{3}{4}y[k-1] + y[k-2] - \frac{5}{16}y[k-3] = u[k-1] - \frac{1}{6}u[k-2] - \frac{1}{2}u[k-3]$$

   (a) (5 points) Compute the transfer function $H(z)$ of the system.

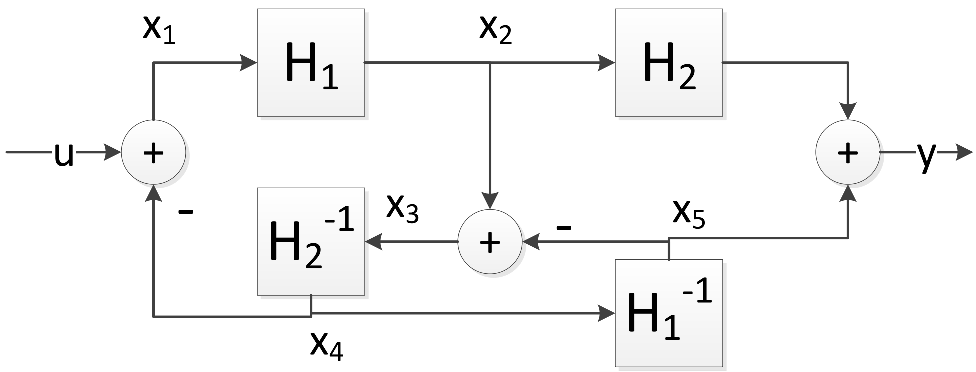
   (b) (5 points) Determine the poles of the system. Hint: one pole is located at $z = \frac{1}{4}$. What does this mean for stability of the system? (Explain your answer!)

   (c) (15 points) Determine the time-domain output $y[k]$ of the system for the input $u[k] = (\frac{1}{3})^k$ given that the system starts completely at rest at $k = 0$ (zero initial conditions)

3. (a) (5 points) A sinusoid $u(t) = \cos(3t)$ is used as as an input to a system $H(s) = \frac{s^2-5s+2}{s^3+6s^2+11s+6}$. Compute the frequency response to the frequency matching the input. Don't leave complex numbers in the denominator.

   (b) (10 points) Given the aforementioned input $u(t) = \cos(3t)$, compute the steady-state time-domain output $y_s(t)$. If you don't have a calculator just specify the angle in terms of the appropriate trigonometric function.

   (c) (15 points) Consider the diagram below.



   Compute the transfer function $H(s)$ of the entire network in terms of the transfer functions $H_1(s)$ and $H_2(s)$.

4. Consider the continuous-time system, given by the state-space representation

$$x'(t) = \begin{pmatrix} 15 & 74 & -120 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} x(t) + \begin{pmatrix} \frac{1}{4} \\ 0 \\ 0 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} x(t) + 2u(t)$$

(a) (15 points) Compute the transfer function $H(s)$ of the system. Do not forget to account for pole-zero cancellation when applicable.

(b) (25 points) Determine a state feedback law $u(t) = -Lx(t)$ such that the poles of the resulting closed loop system are located at respectively -1, -2 and -2.