Mock Exam Simulation and Statistical Analysis

— *Do not turn this page before the official start of the exam!* —

First Name, Surname: ______________________

Student ID: ______________________

Program: Bachelor Data Science and Artificial Intelligence

Course code: KEN2530

Examiner: Dr. J.M.H. Karel & Prof. dr. A.M. Wilbik

Date/time:

Format: Closed book exam

Allowed aides: Pens, Calculator from the DKE list of allowed calculators.

Instructions to students:

- The exam consists of 7 questions on 16 pages (excluding the 1 cover page(s)).
- Fill in your name and student ID number on each page, including the cover page.
- Answer every question at the reserved space below the questions. If you run out of space, continue on the back side, and if needed, use the extra blank page.
- Ensure that you properly motivate your answers.
- Do not use red pens, and write in a readable way. Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- **Good luck!**

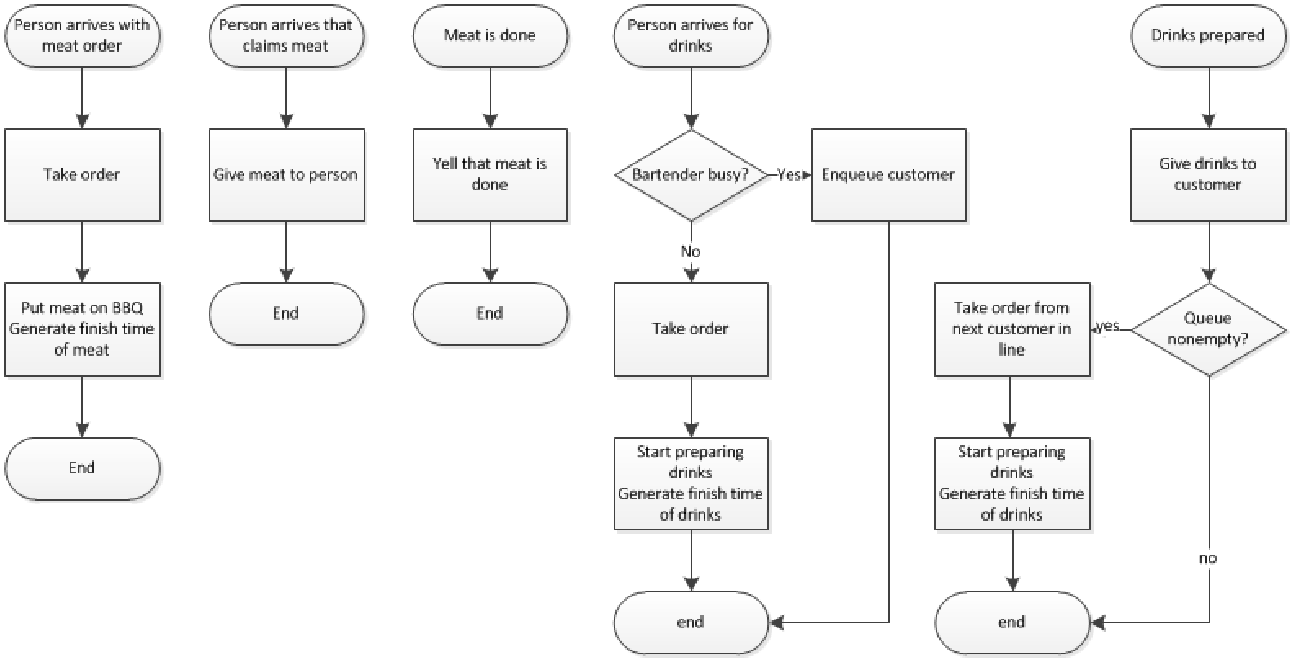
The following table will be filled by the examiner:

Question:	1	2	3	4	5	6	7	Total
Points:	20	20	13	10	10	12	15	100
Score:								

1. True/false questions
   Indicate whether the statement is true or false and argue why. This argument is the most important part of your answer and has the biggest impact on the points earned.

   (a) (5 points) Given that incognito holds a BBQ. Since they want to keep things small they have only one person handling the drinks who handles matters from order to order, hence he only pays attention to the next person in line once the previous one has been served. The person in charge of the meat has an infinitely sized BBQ and will throw on any meat ordered and yell that "the meat is done" once he figures that is about done. If someone then claims it, he will give the meat to that person. The person in charge of the BBQ is so efficient that his actions are timeless.

   A modeller created the following discrete-event model:



   The claim is that this is a valid model for the situation at hand. State whether this claim is true or false and explain why.

   ◯ True
   ◯ False

   Motivation:

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(b) (5 points) If per replication i a single random variable X_i is observed, and these random variables do not follow a normal distribution, but some other symmetric IID distribution, then despite having a large number of replications (e.g. n=10000), it is still not a valid approach to construct a t-confidence interval about the mean since the observations are not normally distributed.

◯ True

◯ False

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(c) (5 points) For the acceptance-rejection method, the majorizing function $t(x)$ must be larger than or equal to the distribution function $F(x)$.

◯ True
◯ False

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(d) (5 points) A runs test is a test, that considers runs of consecutive values satisfying some criterion, in order to test whether the observed values are independent

◯ True
◯ False

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2. Poisson processes A group of three students go for a racing weekend to the Nürburgring and encounter a lot of interesting problems...

(a) (5 points) During their trip they stop at a gas station with two gas pumps and at each of them there is one vehicle filling up and another in queue. For some reason, that only suits your examiner, the time to fill up a car (and pay in the process) is exponentially distributed with mean 4 minutes. However in one of the queues there is a camper at the end of the queue for which the time to fill it up is exponentially distributed with mean 7 minutes. Hence the students choose the other queue. What is the probability that they have chosen the fastest queue?

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(b) (5 points) After arriving at the Nürburgring the students position themselves somewhere alongside the track. Cars pass according to a Poisson process at rate 20 per hour. What is the probability that at least two cars pass in the first five minutes?

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(c) (10 points) At night the students go to a hotel and have a single hotel room with a bathroom and a separate toilet. Going to the toilet has an exponentially distributed duration with mean 3 minutes, going to the bathroom has the same distribution, but with mean 13 minutes. At 22:08 one student already went to the toilet and is using the bathroom and the other two decide that they will each first go to the toilet once vacant and then to the bathroom. Mind you they won't be doing that together. What is the expected time that all students are ready to go to bed?

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3. (a) (10 points) Someone want to take two simple LCGs and combine them. For that he created the following composition LCG:

$$Z_i = (2Z_{i-1} + 4)(\text{mod } 12), \quad \text{for } i \text{ odd} \tag{1}$$
$$Z_i = (3Z_{i-1} + 7)(\text{mod } 12), \quad \text{for } i \text{ even.} \tag{2}$$

Show what the maximum cycle length is that this LCG can obtain, when starting with a seed $Z_0 \in \{0, 1, 2, 3, 4, 5\}$. Hence show it for all those six possible seeds.

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(b) (3 points) Find Z_{111} in case one starts with $Z_0 = 7$.

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4. (10 points) Consider the following pdf:

$$f(x) = \begin{cases} \frac{1}{3}\cos(x) & \text{if } -\frac{1}{2}\pi \leq x < 0 \\ \frac{1}{4\sqrt{x}} & \text{if } 0 \leq x < 1 \\ x^2 & \text{if } 1 \leq x \leq \sqrt[3]{\frac{3}{2}} \\ 0 & \text{otherwise} \end{cases}$$

Give the *inverse transformation* method for generating random variates from this distribution.

5. (10 points) Given the following observations where the first five observations are in the first column, and given that the mean is 0:

-1.4023	1.4193	0.6966	-1.1480	0.1873	0.8404	-0.6003	-2.1384
-1.4224	0.2916	0.8351	0.1049	-0.0825	-0.8880	0.4900	-0.8396
0.4882	0.1978	-0.2437	0.7223	-1.9330	0.1001	0.7394	1.3546
-0.1774	1.5877	0.2157	2.5855	-0.4390	-0.5445	1.7119	-1.0722
-0.1961	-0.8045	-1.1658	-0.6669	-1.7947	0.3035	-0.1941	0.9610

Perform a test ($\alpha = 0.05$) to test whether these observations are independent. What are your conclusions?

6. For a project students have developed two algorithms. For each algorithm a score is calculated (higher is better) and from experience it is known that the score is normally distributed. Each algorithm is independently run 20 times and the average score of the first algorithm is $\bar{X}_1(20) = 67.34$ and the observations have a standard deviation of $S_1(20) = 5.29$. The average score of the second algorithm is $\bar{X}_2(20) = 63.37$ and the observations have a standard deviation of $S_2(20) = 3.92$.

   (a) (6 points) Construct a 95% confidence interval on the mean difference of the scores between the two algorithms. What is your conclusion?

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   (b) (6 points) In case you do not care about differences smaller than $d^* = 0.5$ and you want to select the best algorithm with a certainty of $P^* = 0.95$ how many additional replications of each algorithm do you need?

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7. (15 points) Consider the following Monte Carlo example where U represents a random number:

$$X_{1j} = 1 - 3U \text{ and } X_{2j} = 2U + 3$$

Analytically determine $\mathrm{Cov}(X_{1j}, X_{2j})$ and analytically calculate $\mathrm{Var}(X_{1j} - X_{2j})$ under both independent sampling and common random numbers. What are your conclusions?

Formula sheet Simulation and Statistical Analysis

Basic Statistics

Marginal probability density function of X

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)dy$$

Mean

$$\mu_i = \int_{-\infty}^{\infty} x f_{X_i}(x)dx$$

Sample mean

$$\bar{X}(n) = \frac{\sum_{i=1}^{n} X_i}{n}$$

Variance

$$\sigma_i^2 = E\left[(X_i - \mu_i)^2\right] = E\left(X_i^2\right) - \mu_i^2$$

Sample variance

$$S^2(n) = \frac{\sum_{i=1}^{n} \left[X_i - \bar{X}(n)\right]^2}{n-1}$$

Variance sum law

$$\mathrm{Var}\left[X \pm Y\right] = \mathrm{Var}\left[X\right] + \mathrm{Var}\left[Y\right] \pm 2\mathrm{Cov}\left(X,Y\right)$$

Covariance

$$\mathrm{Cov}(X_i, X_j) = C_{ij} = E\left[(X_i - \mu_i)(X_j - \mu_j)\right]^2 = E\left(X_i X_j\right) - \mu_i \mu_j$$

Correlation

$$\rho_{ij} = \frac{C_i j}{\sqrt{\sigma_i^2 \sigma_j^2}}$$

$100(1-\alpha)\%$ z-confidence about the mean

$$\bar{X}(n) \pm z_{1-\alpha/2}\sqrt{\frac{\sigma^2}{n}}$$

$100(1-\alpha)\%$ t-confidence about the mean

$$\bar{X}(n) \pm t_{n-1,1-\alpha/2}\sqrt{\frac{S^2(n)}{n}}$$

Queueing Systems

Waiting time of i^{th} customer

$$W_i = D_i + S_i$$

Steady-state average delay

$$d = \lim_{n\to\infty} \frac{\sum_{i=1}^{n} D_i}{n} \quad \text{w.p. } 1$$

Steady-state waiting time

$$w = \lim_{n\to\infty} \frac{\sum_{i=1}^{n} W_i}{n} \quad \text{w.p. } 1$$

Steady-state time-average number in queue

$$Q = \lim_{t\to\infty} \frac{\int\limits_0^T Q(t)dt}{T} \quad \text{w.p. } 1$$

Steady-state time-average number in system

$$L = \lim_{t\to\infty} \frac{\int\limits_0^T L(t)dt}{T} \quad \text{w.p. } 1$$

Conservation equations

$$Q = \lambda d \quad \text{and} \quad L = \lambda w$$

Probability Distributions

Uniform density

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

Uniform distribution

$$F(x) = \begin{cases} 0 & \text{if } x < a \\ \frac{x-a}{b-a} & \text{if } a \le x \le b \\ 1 & \text{if } b < x \end{cases}$$

Exponential density

$$f(x) = \begin{cases} \frac{1}{\beta} e^{-x/\beta} & \text{if } x \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

Exponential distribution

$$f(x) = \begin{cases} 1 - e^{-x/\beta} & \text{if } x \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

Normal density

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Poisson mass

$$p(x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!} & \text{if } x \in \{0, 1, \ldots\} \\ 0 & \text{otherwise} \end{cases}$$

Poisson distribution

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ e^{-\lambda} \sum\limits_{i=1}^{\lfloor x \rfloor} \frac{\lambda^i}{i!} & \text{if} x \ge 0 \end{cases}$$

Empirical tests of Hypothesis

χ^2 test

$$\chi^2 = \sum_{j=1}^{k} \frac{(N_j - np_j)^2}{np_j}$$

Kolmogorov-Smirnov test

Case	Adjusted test statistic	0.850	0.900	0.950	0.975	0.990
All parameters known	$\left(\sqrt{n} + 0.12 + \frac{0.11}{\sqrt{n}}\right) D_n$	1.138	1.224	1.358	1.480	1.628
$\mathrm{N}\left(\bar{X}(n), S^2(n)\right)$	$\left(\sqrt{n} - 0.01 + \frac{0.85}{\sqrt{n}}\right) D_n$	0.775	0.819	0.895	0.955	1.035
$\mathrm{expo}\left(\bar{X}(n)\right)$	$\left(D_n - \frac{0.2}{n}\right)\left(\sqrt{n} + 0.26 + \frac{0.5}{\sqrt{n}}\right)$	0.926	0.990	1.094	1.190	1.308

Poker test

Pattern	Probability
all different	0.3024
one pair	0.5040
two pair	0.1080
three of a kind	0.072
full house	0.009
Four of a kind	0.0045
five of a kind	0.0001

Runs test

$$\mu_r = \frac{2ab}{n} + 0.5$$
$$\sigma_r^2 = \frac{2ab(2ab-n)}{n^2(n-1)}$$

Comparing Alternative System Configurations

Estimated degrees of freedom for Welch's confidence interval

$$\hat{f} = \frac{\left[\frac{S_1^2(n_1)}{n_1} + \frac{S_2^2(n_2)}{n_2}\right]^2}{\frac{\left[\frac{S_1^2(n_1)}{n_1}\right]^2}{n_1-1} + \frac{\left[\frac{S_2^2(n_2)}{n_2}\right]^2}{n_2-1}}$$

Dudewics and Dalal sample size

$$N_i = \max\left\{n_0 + 1, \left\lceil \frac{h_1^2 S_i^2(n_0)}{(d^*)^2} \right\rceil\right\}$$

Dudewics and Dalal weights

$$W_{i1} = \frac{n_0}{N_i}\left\{1 + \sqrt{1 - \frac{N_i}{n_0}\left[1 - \frac{(N_i - n_0)(d^*)^2}{h_1^2 S_i^2(n_0)}\right]}\right\}$$

Dudewics and Dalal table

$\mathbf{P}^*$	n_0	$k=2$	$k=3$	$k=4$
0.90	20	1.896	2.342	2.583
0.90	40	1.852	2.283	2.514
0.95	20	2.453	2.872	3.101
0.95	40	2.386	2.786	3.003

Variance Reduction

Common random numbers

$$\mathrm{Var}[\bar{Z}(n)] = \frac{\mathrm{Var}(X_{1j}) + \mathrm{Var}(X_{2j}) - 2\mathrm{Cov}(X_{1j}, X_{2j})}{n}$$

Antithetic variables

$$\mathrm{Var}[\bar{X}(n)] = \frac{\mathrm{Var}(X_j^{(1)}) + \mathrm{Var}(X_j^{(2)}) + 2\mathrm{Cov}(X_j^{(1)}, X_j^{(2)})}{4n}$$

Control variates

$$\begin{aligned} \mathrm{Var}(X_c) &= \mathrm{Var}(X) + a^2\mathrm{Var}(Y) - 2a\mathrm{Cov}(X, Y) \\ a^* &= \frac{\mathrm{Cov}(X, Y)}{\mathrm{Var}(Y)} \end{aligned}$$

Critical points for the Student t-distribution

Qnt	0.7500	0.8000	0.8500	0.9000	0.9500	0.9750	0.9900	0.9950	0.9975	0.9990	0.9995
1 dof	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6
2 dof	0.816	1.080	1.386	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60
3 dof	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92
4 dof	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5 dof	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6 dof	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7 dof	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8 dof	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9 dof	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10 dof	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11 dof	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12 dof	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13 dof	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14 dof	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15 dof	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16 dof	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17 dof	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18 dof	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19 dof	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20 dof	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21 dof	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22 dof	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23 dof	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24 dof	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25 dof	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26 dof	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27 dof	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28 dof	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29 dof	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30 dof	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40 dof	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
50 dof	0.679	0.849	1.047	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496
60 dof	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
80 dof	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416
100 dof	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
120 dof	0.677	0.845	1.041	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞ dof	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Critical points for the χ^2-distribution

Quantile	0.05	0.10	0.20	0.30	0.50	0.70	0.80	0.90	0.95	0.99	0.999
1 dof	0.004	0.02	0.06	0.15	0.46	1.07	1.64	2.71	3.84	6.64	10.83
2 dof	0.10	0.21	0.45	0.71	1.39	2.41	3.22	4.60	5.99	9.21	13.82
3 dof	0.35	0.58	1.01	1.42	2.37	3.66	4.64	6.25	7.82	11.34	16.27
4 dof	0.71	1.06	1.65	2.20	3.36	4.88	5.99	7.78	9.49	13.28	18.47
5 dof	1.14	1.61	2.34	3.00	4.35	6.06	7.29	9.24	11.07	15.09	20.52
6 dof	1.63	2.20	3.07	3.83	5.35	7.23	8.56	10.64	12.59	16.81	22.46
7 dof	2.17	2.83	3.82	4.67	6.35	8.38	9.80	12.02	14.07	18.48	24.32
8 dof	2.73	3.49	4.59	5.53	7.34	9.52	11.03	13.36	15.51	20.09	26.12
9 dof	3.32	4.17	5.38	6.39	8.34	10.66	12.24	14.68	16.92	21.67	27.88
10 dof	3.94	4.87	6.18	7.27	9.34	11.78	13.44	15.99	18.31	23.21	29.59

Normal distribution

z	0	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09
0	0,5	0,50399	0,50798	0,51197	0,51595	0,51994	0,52392	0,5279	0,53188	0,53586
0,1	0,53983	0,5438	0,54776	0,55172	0,55567	0,55962	0,56356	0,56749	0,57142	0,57535
0,2	0,57926	0,58317	0,58706	0,59095	0,59483	0,59871	0,60257	0,60642	0,61026	0,61409
0,3	0,61791	0,62172	0,62552	0,6293	0,63307	0,63683	0,64058	0,64431	0,64803	0,65173
0,4	0,65542	0,6591	0,66276	0,6664	0,67003	0,67364	0,67724	0,68082	0,68439	0,68793
0,5	0,69146	0,69497	0,69847	0,70194	0,7054	0,70884	0,71226	0,71566	0,71904	0,7224
0,6	0,72575	0,72907	0,73237	0,73565	0,73891	0,74215	0,74537	0,74857	0,75175	0,7549
0,7	0,75804	0,76115	0,76424	0,7673	0,77035	0,77337	0,77637	0,77935	0,7823	0,78524
0,8	0,78814	0,79103	0,79389	0,79673	0,79955	0,80234	0,80511	0,80785	0,81057	0,81327
0,9	0,81594	0,81859	0,82121	0,82381	0,82639	0,82894	0,83147	0,83398	0,83646	0,83891
1	0,84134	0,84375	0,84614	0,84849	0,85083	0,85314	0,85543	0,85769	0,85993	0,86214
1,1	0,86433	0,8665	0,86864	0,87076	0,87286	0,87493	0,87698	0,879	0,881	0,88298
1,2	0,88493	0,88686	0,88877	0,89065	0,89251	0,89435	0,89617	0,89796	0,89973	0,90147
1,3	0,9032	0,9049	0,90658	0,90824	0,90988	0,91149	0,91309	0,91466	0,91621	0,91774
1,4	0,91924	0,92073	0,9222	0,92364	0,92507	0,92647	0,92785	0,92922	0,93056	0,93189
1,5	0,93319	0,93448	0,93574	0,93699	0,93822	0,93943	0,94062	0,94179	0,94295	0,94408
1,6	0,9452	0,9463	0,94738	0,94845	0,9495	0,95053	0,95154	0,95254	0,95352	0,95449
1,7	0,95543	0,95637	0,95728	0,95818	0,95907	0,95994	0,9608	0,96164	0,96246	0,96327
1,8	0,96407	0,96485	0,96562	0,96638	0,96712	0,96784	0,96856	0,96926	0,96995	0,97062
1,9	0,97128	0,97193	0,97257	0,9732	0,97381	0,97441	0,975	0,97558	0,97615	0,9767
2	0,97725	0,97778	0,97831	0,97882	0,97932	0,97982	0,9803	0,98077	0,98124	0,98169
2,1	0,98214	0,98257	0,983	0,98341	0,98382	0,98422	0,98461	0,985	0,98537	0,98574
2,2	0,9861	0,98645	0,98679	0,98713	0,98745	0,98778	0,98809	0,9884	0,9887	0,98899
2,3	0,98928	0,98956	0,98983	0,9901	0,99036	0,99061	0,99086	0,99111	0,99134	0,99158
2,4	0,9918	0,99202	0,99224	0,99245	0,99266	0,99286	0,99305	0,99324	0,99343	0,99361
2,5	0,99379	0,99396	0,99413	0,9943	0,99446	0,99461	0,99477	0,99492	0,99506	0,9952
2,6	0,99534	0,99547	0,9956	0,99573	0,99585	0,99598	0,99609	0,99621	0,99632	0,99643
2,7	0,99653	0,99664	0,99674	0,99683	0,99693	0,99702	0,99711	0,9972	0,99728	0,99736
2,8	0,99744	0,99752	0,9976	0,99767	0,99774	0,99781	0,99788	0,99795	0,99801	0,99807
2,9	0,99813	0,99819	0,99825	0,99831	0,99836	0,99841	0,99846	0,99851	0,99856	0,99861
3	0,99865	0,99869	0,99874	0,99878	0,99882	0,99886	0,99889	0,99893	0,99896	0,999
3,1	0,99903	0,99906	0,9991	0,99913	0,99916	0,99918	0,99921	0,99924	0,99926	0,99929
3,2	0,99931	0,99934	0,99936	0,99938	0,9994	0,99942	0,99944	0,99946	0,99948	0,9995
3,3	0,99952	0,99953	0,99955	0,99957	0,99958	0,9996	0,99961	0,99962	0,99964	0,99965
3,4	0,99966	0,99968	0,99969	0,9997	0,99971	0,99972	0,99973	0,99974	0,99975	0,99976
3,5	0,99977	0,99978	0,99978	0,99979	0,9998	0,99981	0,99981	0,99982	0,99983	0,99983
3,6	0,99984	0,99985	0,99985	0,99986	0,99986	0,99987	0,99987	0,99988	0,99988	0,99989
3,7	0,99989	0,9999	0,9999	0,9999	0,99991	0,99991	0,99992	0,99992	0,99992	0,99992
3,8	0,99993	0,99993	0,99993	0,99994	0,99994	0,99994	0,99994	0,99995	0,99995	0,99995
3,9	0,99995	0,99995	0,99996	0,99996	0,99996	0,99996	0,99996	0,99996	0,99997	0,99997
4	0,99997	0,99997	0,99997	0,99997	0,99997	0,99997	0,99998	0,99998	0,99998	0,99998