Exam **Mathematical Simulation**
Bachelor Knowledge Engineering, block 2.5
Tuesday 2 July 2009, 14.00-17.00h, Bouillonstraat 8-10, room 0.015
Please make sure that your answers are to-the-point and concise, but ensure that you give a **proper explanation**! *Ensure that your cell phone is turned off and is not within reach!*

1. In the absence of his wife, a father has to take care of their twin babies. He has worked out some sort of system to do this. If a baby starts crying he takes it out of the crib and checks whether it wants to drink milk or whether the diaper needs to be changed. In the former case the father puts the baby back in the crib, fetches two bottles of warm milk, and first feeds the baby that started crying and then the other one. In the latter case he only changes the diaper of the crying baby. Then the father sits on the couch until the next time that one of the babies starts crying. A baby does not cry if it is being fed or having a diaper changed and if the other baby starts crying during one of these activities the father ignores that until he has time to attend to this baby.
   (a) Give an overview of the activities with their corresponding component(class) and the state variables that might be changed by that process.
   (b) Draw the diagrams of a process based description of the described situation.

2. A car dealer selling Hummers has two salesmen. On average a smooth sales talk takes 13 minutes and the time it takes is exponentially distributed. Customers that want to buy Hummers arrive according to a Poisson process at rate 3 per hour.
   (a) Three customers, conveniently named Mr. Custone, Mrs. Custtwo and Ms. Custthree enter the showroom simultaneously. Really needing a Hummer Mr. Custone and Mrs. Custtwo go to the salesmen directly and Ms. Custthree waits in the showroom until a salesman has time to sell her a car. What is the probability that Mr. Custone is still talking to the salesmen after Ms. Custthree drives away in her new Hummer?
   (b) Due to the recession the car dealer decides to sell a slightly different car as well: the Smart, such that the customers have more choice. It is expected that demand for Hummers remains the same, but that in addition customers for the Smart arrive according to a Poisson process at rate 13 per hour. The owner figures that he has to hire extra salesmen. All salesmen can sell both types of car and basically give the same sales talk for both cars, requiring the same amount of time. The owner hires as many salesmen as possible under the restriction that the utilization rate of the salesmen is no less then 0.80. How many extra salesmen will he hire?
   (c) Given a time interval of a certain (long) length, describe two methods to generate arrival times of Poisson arrivals with rate λ in that interval.

3. Consider the following LCG with multiplier $a = 5$, increment $c = 0$, modulus $m = 16$ and seed $Z_0 = 1$.
   (a) Determine the period of this LCG.
   (b) Determine the value of Z_{1025}.
   (c) Describe how a single LCG with sufficiently long period can be used to produce multiple streams of random numbers.

4. (a) Describe an approach to use the inverse transformation method for generating random variates from a distribution for which no explicit inverse exists.
   (b) Consider the following density:

$$f(x) = \begin{cases} \frac{5x^4}{2} & \text{if } -1 \leq x \leq 0 \\ 6x^3 & \text{if } 0 < x \leq \sqrt[4]{\frac{1}{3}} \\ 0 & \text{otherwise} \end{cases}$$

Give the *inverse-transform* algorithm for generating random variates from this distribution.

5. Given the following table with random numbers, where the first five numbers are in the first column, etc.:

0.0040	0.6245	0.5618	0.9611	0.5597	0.1858	0.0849	0.8574	0.1464	0.1727
0.3471	0.2695	0.4377	0.0370	0.5159	0.1175	0.0094	0.3328	0.7850	0.9930
0.8536	0.0733	0.2361	0.5427	0.6019	0.7645	0.7800	0.3140	0.0690	0.6591
0.9265	0.4672	0.5173	0.9044	0.0358	0.7945	0.6485	0.5079	0.8738	0.0621
0.7803	0.6852	0.0622	0.9567	0.0240	0.4903	0.9457	0.9922	0.0844	0.0442

Perform a runs test to determine whether you may reject the null hypothesis that these are random independent draws from the uniform $(0, 1)$ distribution. What are your conclusions?

6. (a) Explain what the batch means method is used for.

   (b) A pirate captain has developed a strategy on which ships to capture. In a trial of his strategy he independently seizes 15 ships, yielding a loot of respectively €412,000.-, €386,000.- €450,000.-, €358,000.-, €552,000.-, €390,000.- €407,000.-, €474,000.-, €404,000.-, €393,000.-, €341,000.-, €420,000.-, €306,000.-, €450,000.- and €513,000.-. Construct a 90% *confidence interval* for the loot.

   (c) The captain's parrot disagrees with him and comes up with a different strategy. The crew also test this strategy by seizing 13 ships yielding an average loot of €444,461.50 and a sample variance of 225,269,230. Can you conclude using a 90% confidence interval that either strategy is better? Explain!

7. A chairman of a polling station for the European elections wants to ensure that things go smoothly in his polling station on June 4th. He has worked three set-ups for his polling station which he each simulated 40 times, measuring the average delay of the voters. The mean and variance over the 40 replications of the first configuration is: $\bar{X}_1^{(1)}(40) = 3.0378$ and $S_1^2(40) = 2.102$, for the second configuration: $\bar{X}_2^{(1)}(40) = 4.4381$ and $S_2^2(40) = 2.4857$ and for the third configuration: $\bar{X}_3^{(1)}(40) = 3.4836$ and $S_3^2(40) = 2.9472$.

   (a) How many additional replications of each system are needed in order to select the best system, with $P^* = 0.95$ and $d^* = 0.2$, using the algorithm of *Dudewicz and Dalal*?

   (b) Suppose that the additional replication as required under (a) are performed and the resulting second-stage sample means are $\overline{X}_1^{(2)} = 3.2756$, $\overline{X}_2^{(2)} = 4.4735$ and $\overline{X}_3^{(2)} = 3.5876$. Can the chairman now select the *best configuration* and, if "yes", what configuration should he select?

8. (a) Consider the following Monte Carlo example where U represents a random number:

$$X_j = 3U^2 - 1\frac{1}{3}$$

   Analytically determine $\mathrm{Cov}(X_j^{(1)}, X_j^{(2)})$ and analytically calculate $\mathrm{Var}[(X_j^{(1)} + X_j^{(2)})/2]$ under both independent sampling and antithetic variables. What are your conclusions?

   (b) Under what conditions should you use antithetic variables instead of common random numbers?

Success!