

Exam Mathematical Simulation
 Bachelor Knowledge Engineering, block 2.5
 Friday July 1st, 2016, 9.00-12.00h

Please make sure that your answers are to-the-point and concise, but ensure that you give
 a **proper explanation!** *Ensure that you adhere to the exam regulations!*

This is an open book exam. Try to answer the questions at the reserved spaces in the
 assignments sheet. The total number of points is 100

Name: _____

Student ID: _____

1. State whether the following propositions are true or false. A good explanation is essential

(a) (5 points) When comparing the mean outcome of three alternative system configuration by all-pairwise comparisons, one can assess the significance at α of these results by constructing three $100(1 - \frac{\alpha}{3})\%$ confidence intervals on the pairwise differences in means.

- ☐ True
- ☐ False

Explanation:

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(b) (5 points) In the case of Poisson arrivals with a time-varying arrival rate, it would be an acceptable approximation to use the instantaneous arrival rate, when determining the next interarrival time, in the case that the time-varying arrival rates are always high. This probably would give results similar to the thinning algorithm.

- ☐ True
- ☐ False

Explanation:

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(c) (5 points) In a discrete-event simulation program the timing routine keeps track of the computer time that a simulation uses, such that the performance of the program can be optimized.

- ☐ True
- ☐ False

Explanation:

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- (d) (5 points) If the inverse of a cumulative distribution function has no closed form solution, then the inverse transformation method for generating random variates cannot be used.

- ☐ True
☐ False

Explanation:

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2. At an airport coffee corner, travelers buy hot drinks at a rate of 9 per minute. Their choice is between coffee, tea, and hot chocolate. The orders for each kind of hot drink happen to arrive according to independent Poisson processes. The mean interarrival time between orders for tea is 21 seconds. The rate by which hot chocolate gets ordered, is 1 per 7 minutes.

- (a) (4 points) What is the mean interarrival time for orders of coffee?

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- (b) (4 points) The manager of the coffee corner notes that it is now 15h:00m:00s. What is the expected time for the next order of hot chocolate?

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- (c) (6 points) If the average service time (including serving, payment, cleaning up, and so on) for each cup of coffee or tea is 30 seconds, while for hot chocolate it is 60 seconds, then at least how much personnel is needed to avoid a queue to build up permanently? (Explain!)

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3. (a) (7 points) Given the following LCG:

$$Z_i = (2Z_{i-1} + 6)(\text{mod}13)$$

Map all cycles of this LCG

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- (b) (5 points) Give an example of an LCG and a seed in the range of the LCG, for which this seed does not lie on any cycle of the LCG.

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4. Consider the following pdf:

$$f(x) = \begin{cases} \frac{3}{16}x^2 & \text{if } -2 \leq x < -1 \\ \frac{3}{8}x^4 & \text{if } -1 \leq x < 1 \\ \frac{1}{x^2} & \text{if } 1 \leq x \leq a \\ 0 & \text{otherwise} \end{cases}$$

(a) (6 points) Find the corresponding cumulative distribution function $F(x)$.

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(b) (4 points) Find a such that $F(x)$ is a valid distribution function.

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- (c) (7 points) Give the *inverse transformation* method for generating random variates from this distribution.

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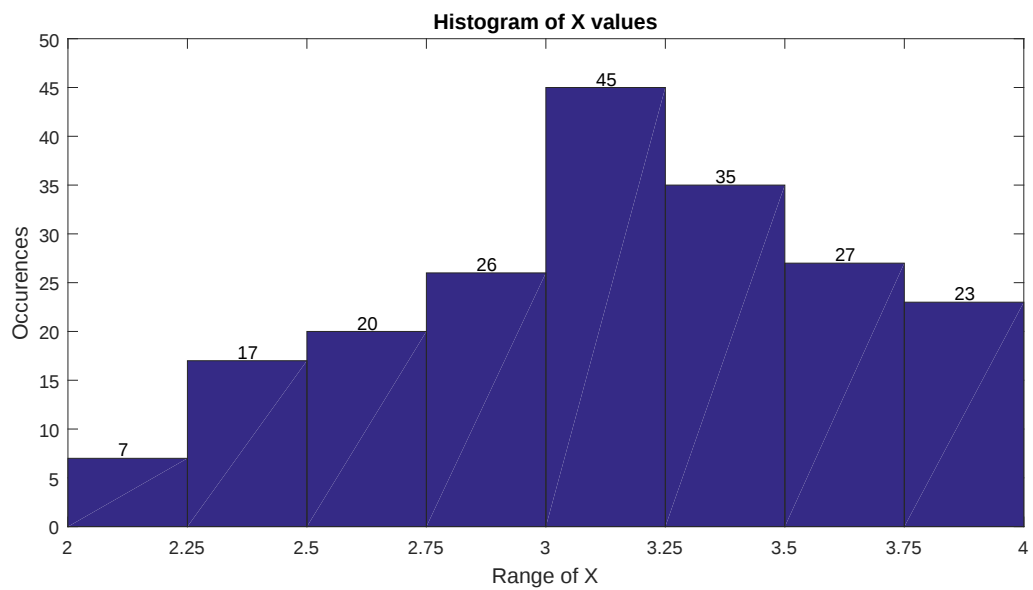
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5. (12 points) Someone is running a simulation for which the results obtained are unexpected. Looking into the details the suspicion arises that the $U(2, 4)$ random variates X that are being generated are wrong. This is visually observed in the histogram:



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6. Someone wants to determine a confidence interval of the mean of the waiting time of customers in a simulation. For that 10 simulation runs are made, for each run the mean waiting time is computed and it is assumed that the runs are long enough such that the mean waiting time is normally distributed. The mean waiting times obtained are: 1.9503, 15.1048, 8.1762, 5.8108, 8.1442, 5.3851, 5.6276, 10.4691, 10.2271 and 10.2516. The mean of these observations is 8.1147 and the variance 13.3757.

(a) (5 points) Determine a 95% confidence interval of the mean waiting time.

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(b) (5 points) Suppose one does not want to make the normality assumption. Describe what steps you can do to determine a 95% confidence interval without making assumptions on the distribution of the means. You do not actually have to compute it, but explain it in an algorithmic way.

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7. (15 points) Consider the following Monte Carlo example where U represents a random number:

$$X_j = U^2 + 3U$$

It is decided to use antithetic variables as a variance reduction technique. Analytically determine $\text{Cov}(X_j^{(1)}, X_j^{(2)})$ and analytically calculate $\text{Var}\left(\frac{X_j^{(1)} + X_j^{(2)}}{2}\right)$ under both independent sampling and antithetic variables. What are your conclusions?

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Success!

Question:	1	2	3	4	5	6	7	Total
Points:	20	14	12	17	12	10	15	100
Score:								