



Talmud

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Contested Garment

Coalitional Procedure

Practice (Ex 16 & 17) Very Important!!

Hydraulic rationing (Visualisation)

Contested Garment

Basically if two people where fighting over a garment, you split the part that is contested between the two equally.

In the below image A claims full and B claims B.



This is called the **CG-principle**.

Coalitional Procedure

How to get a CG-principle consistent solution with more than 2 players?

(400 | 100, 160, 250, 300, 350)

1. In the first step we group claimants C_2, C_3, C_4 and C_5 to be one widow W_1 who claims $c_2 + c_3 + c_4 + c_5 = 160 + 250 + 300 + 350 = 1050$, while C_1 is still claiming $c_1 = 100$. When C_1 and W_1 share the estate $E = 400$ according to the CG-principle, then C_1 receives $x_1 = 50$ and W_1 gets $E_1 = 350$, because C_1 only claims 100 and W_1 claims the full estate.
2. Now we share 350 among widows C_2 , claiming $c_2 = 160$ and widow W_2 , claiming $c_3 + c_4 + c_5 = 250 + 300 + 350 = 900$. The CG-solution gives the amount $x_2 = 80$ for C_2 and 270 for W_2 .
3. The amount 270 is now to be shared among C_3 claiming $c_3 = 250$ and W_3 claiming $c_4 + c_5 = 300 + 350 = 650$. The CG-solution for this two-person problem would be to give 125 to C_3 and 145 to W_3 . This would imply that either C_4 or C_5 is getting less than C_3 , since C_4 and C_5 have to share 145. This would contradict the CG-principle. Therefore C_3, C_4 and C_5 should share the amount 270 equally, that is $x_3 = x_4 = x_5 = 90$.

At this point the procedure ends and the estate of 400 is shared among the widows as (50, 80, 90, 90, 90).

At each step of this procedure, we have to check 2 things;

- 1) remaining estate $\geq$ number of players in coalition $\times$ how much current player got in total
- 2) demanded estate by coalition - remaining estate $\geq$ number of players in coalition $\times$ how much current player got from the 50/50 split only

If condition 1 fails, split the remaining estate equally.

If condition 2 fails, split the losses equally

Condition 1 checks that the members of the coalition are not going to get less (on

average) than the person leaving with a lower claim.

Condition 2 checks that in every step the loss the single claimant is incurring is not bigger than the average loss for the coalition claimants.

In words: small claimants get less than big claimants and small claimants lose less than big claimants.

Practice (Ex 16 & 17) Very Important!!

Problem 16 Use the coalitional procedure to find a solution for the five-person bankruptcy problem $(600 \mid 300, 350, 400, 450, 600)$.

Verify that the solution is CG-consistent.

Coalitional procedure:

	600
300	150
1800	450

$$4 \times 150 \leq 450 \text{ False}$$

$$4 \times 150 \leq 1800 - 450$$

The first check in the first step fails and therefore the answer will be: $(120, 120, 120, 120, 120)$.

Verify:

	240
C_i	120
C_j	120

Problem 17 Use the coalitional procedure to find a solution for the three-person bankruptcy problem $(550 \mid 100, 200, 300)$.

Verify that the solution is CG-consistent.

Coalitional procedure:

	550
100	25+50
500	25+400+50

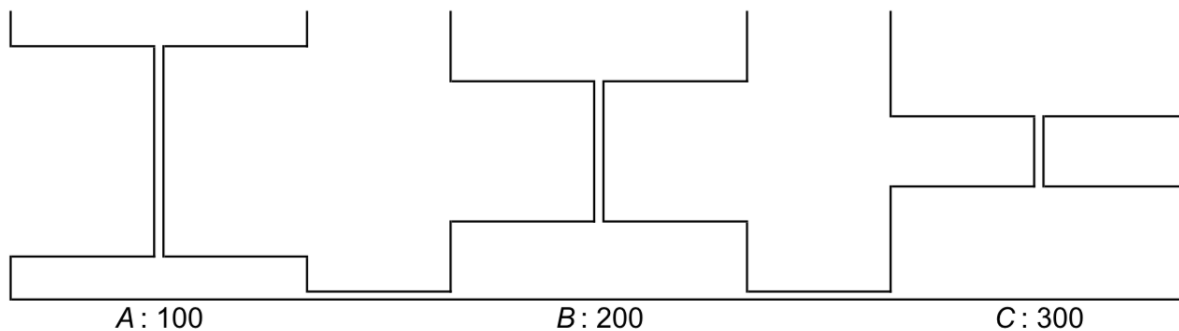
$$2 \times 75 \leq 450$$

$$2 \times 25 \leq 500 - 475 \text{ False}$$

The second check in the first step fails, therefore the answer will be:
 $(100 - 16\frac{2}{3}, 200 - 16\frac{2}{3}, 300 - 16\frac{2}{3}) = (83\frac{1}{3}, 183\frac{1}{3}, 283\frac{1}{3})$ (note: $(\frac{(100+200+300)-550}{3}) = 16\frac{2}{3}$)
 Verify:

	$266\frac{2}{3}$		$366\frac{2}{3}$		$466\frac{2}{3}$
100	$33\frac{1}{3} + 50$	100	$33\frac{1}{3} + 50$	200	$83\frac{1}{3} + 100$
200	$33\frac{1}{3} + 100 + 50$	300	$33\frac{1}{3} + 200 + 50$	300	$83\frac{1}{3} + 100 + 100$

Hydraulic rationing (Visualisation)



A way to visualise CG-consistent solutions.

You do not have to draw actual vessels in the exam, just lines like below are enough.

You need to agree on a certain scale before starting and stick to that when drawing the lines.

The you need to draw a line horizontally representing the estate of the poor dead person. You need to make sure that it is at the right level, its usually easiest to start from the mid point of the claims and then work upward or downwards with the counting (what the red counting on the right represents).... be careful a block does not represent the same number everywhere, for example the green block represents 150 vs purple 400,

this is because there are more vessels absorbing water on the edges compared to the middle.

