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Games in Normal Form

Files	Game Theory September 13 2021.pdf
Link	
Status	Done
Type	Lecture

Nash-Equilibrium:

A pair of actions or strategies that are best replies to each other

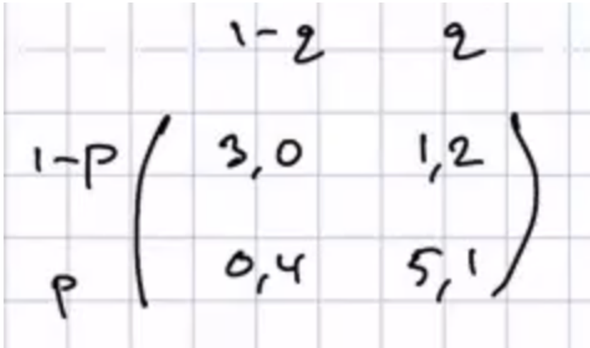
$$ai*j* = maxaij*$$

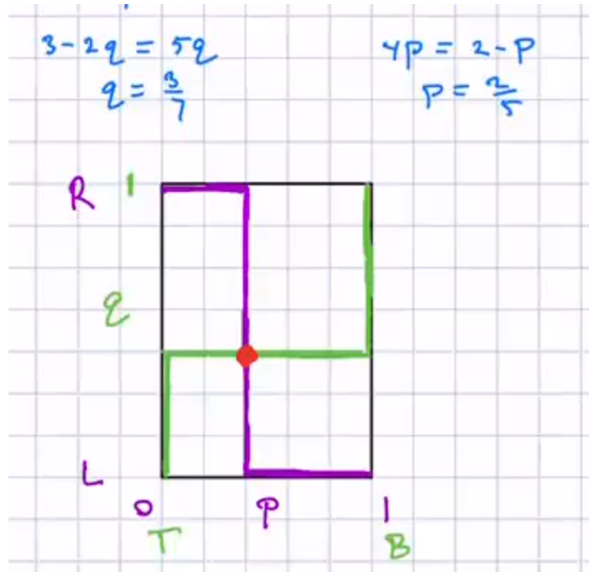
$$bi*j* = maxai*j$$

Nash proved that for any game, as long as there is a finite set of players and actions there is at least one Nash equilibrium. If there is more than one the payoffs can be different at each one!!!

Mixed actions: when a player chooses his actions based on a probability device, represented by a vector

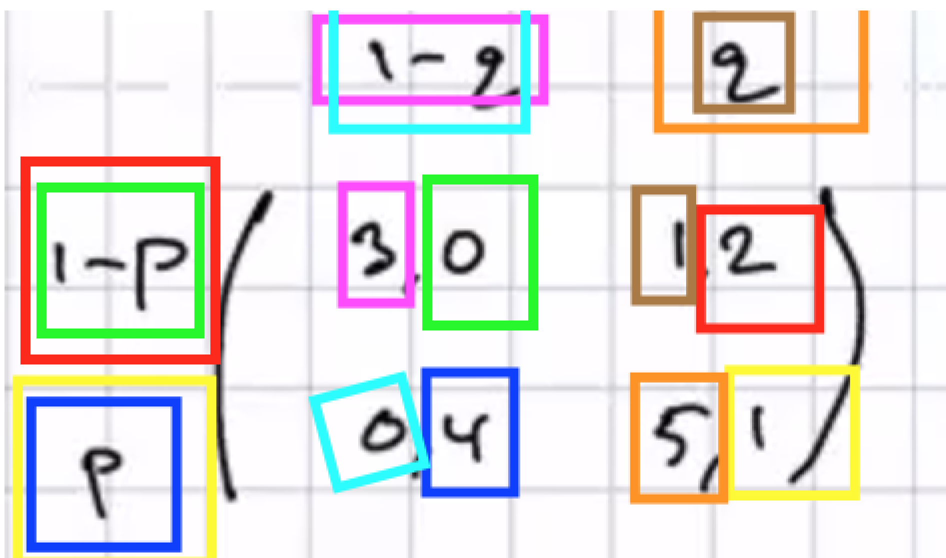
Example of finding nash-equilibrium for a game and drawing the square best reply diagram





To find the equations on the top of the right image;

- (green x green) + (blue x blue) = (red x red) + (yellow x yellow)
- (pink x pink) + (brown x brown) = (cyan x cyan) + (orange x orange)



To find where the p and q should be you solve the system of linear equations, like shown on top of the right picture.

To find where whether the link line should extend to the left or right side at the top and bottom of the rectangle (the p line), we need to look at the payoffs of player 2 (the number on the right in the payoff matrix). If p is 0 then it is best for player 2 to play right, if p is 1 then it is best for payer to to play left. We do a similar thing for q.

The nash equilibrium(s) are all the points where the pink and green line touch (the red point in the middle) but also sometimes they can touch on the edge of the rectangle (not in this example though), you need to list them out in the exam.

Correlated equilibrium

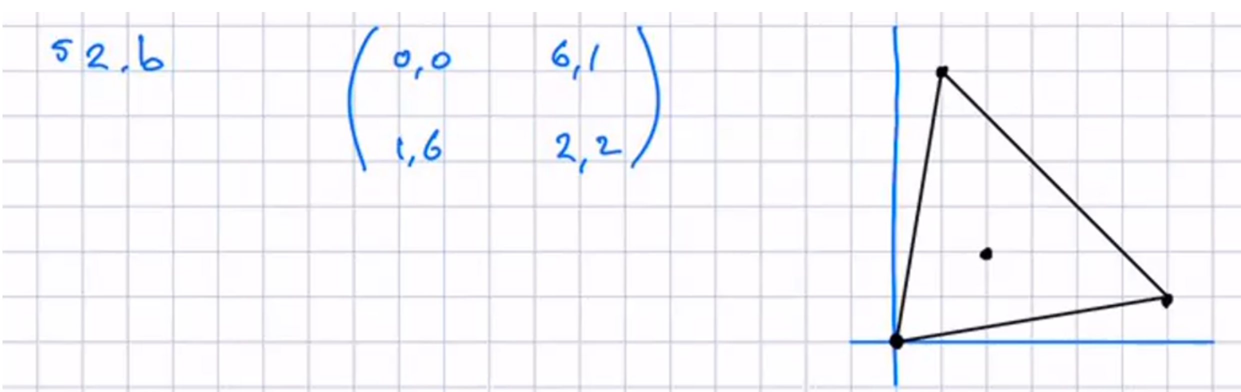
Equilibrium achieved with a correlation device that has stabilising properties, represented by a probability matrix that sums up to 1.

An arbitrator tells players to choose strategies based on the probability device, and it is in the best interest of players to follow the arbitrator advice because these are the best replies.

An example for battle of sexes is provided here, there can be many correlated equilibriums.

$$\begin{pmatrix} 2,1 & 0,0 \\ 0,0 & 1,2 \end{pmatrix} \qquad P^* = \begin{pmatrix} 0.5 & 0.2 \\ 0 & 0.3 \end{pmatrix}.$$

To find out if a certain expected pay off is possible using a correlated device, we need to check if the expected payoff lies within the xy-plane. Example:



You can prove that a device is a correlated equilibrium by checking check every recommendation with both players (top and bottom for player 1, left and right for player 2) and if in every case it is in the each players best interest to actually do what the device recommended then it is a correlated equilibrium.

(Its easy to check if a certain payoff works but its hard to come up with a probability device -Alexander the great)

To find a probability device you need to leverage certain properties in the payoff matrix. For example if an entry is 0,0 you can put any leftover weight there, as the matrix needs to add up to 1. Or you can set one of the variables in the probability matrix to 0 and try to find numbers that give desired outcomes from the other two using a system of linear equations. I do not know a more robust answer but here is an example on doing that:

$$\begin{pmatrix} \square & x \\ y & 0 \end{pmatrix} \begin{pmatrix} 0,0 & 6,1 \\ 1,6 & 2,9 \end{pmatrix}$$

$$6x + y = 3$$

$$x + 6y = 3$$

$$x = 3 - 6y$$

$$18 - 36y + y = 3$$

$$18 - 35y = 3$$

$$15 = 35y$$

$$y = \frac{3}{7}$$

$$x = 3 - 6y$$

$$x = 3 - 6\left(\frac{3}{7}\right)$$

$$x = \frac{21}{7} - \frac{18}{7}$$

$$x = \frac{3}{7}$$

left

$$x + y = \frac{6}{7} \quad \left(\frac{1}{7}\right)$$

$$\begin{pmatrix} \frac{1}{7} & \frac{3}{7} \\ \frac{3}{7} & 0 \end{pmatrix}$$