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# Matrix Games

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| Files | Game Theory September 9 2021.pdf |
| Link | |
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## Matrix Games

In this chapter we tackle zero sum games in which $B = -A$

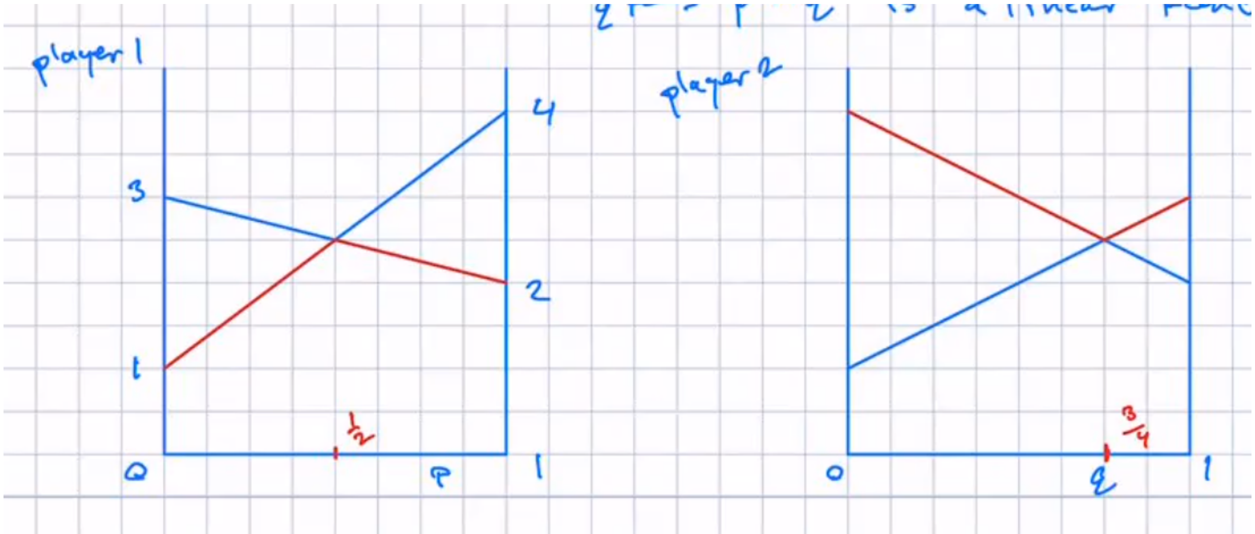
So we only need 1 set of numbers A and we know the payoffs of player 2 as soon as the payoffs of player 1 are known. Player 1 is trying to maximise and player 2 is trying to minimise.

S**addle-point:** when a game has an equilibrium in pure actions

You can find the value if a game by looking for a row such that the minimum value in the row is highest compared to the minimum of all the other rows. Then player 1 will pick this row and player 2 will pick the minimum value of this row since we assume they are both rational players that have full information and are risk neutral.

You can draw a graph to represent the game from the perspective of either player. Example:

$$\begin{array}{cc}
1-p & \multirow{2}{*}{} \\
p &
\end{array}
\begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}$$




$$\begin{array}{ccc}
 & \begin{array}{cc} \frac{1}{4} & \frac{3}{4} \\ 1-q & q \end{array} & \\
\begin{array}{c} \frac{1}{2} \quad 1-p \\ \\ \frac{1}{2} \quad p \end{array} & \begin{pmatrix} 1 & 3 \\ & \\ 4 & 2 \end{pmatrix} & \begin{array}{l} 1+2q \quad 2.5 \\ \\ \qquad\quad 2.5 \end{array} \\
 & \begin{array}{cc} 1+3p & 3-p \\ 2.5 & 2.5 \end{array} &
\end{array}$$

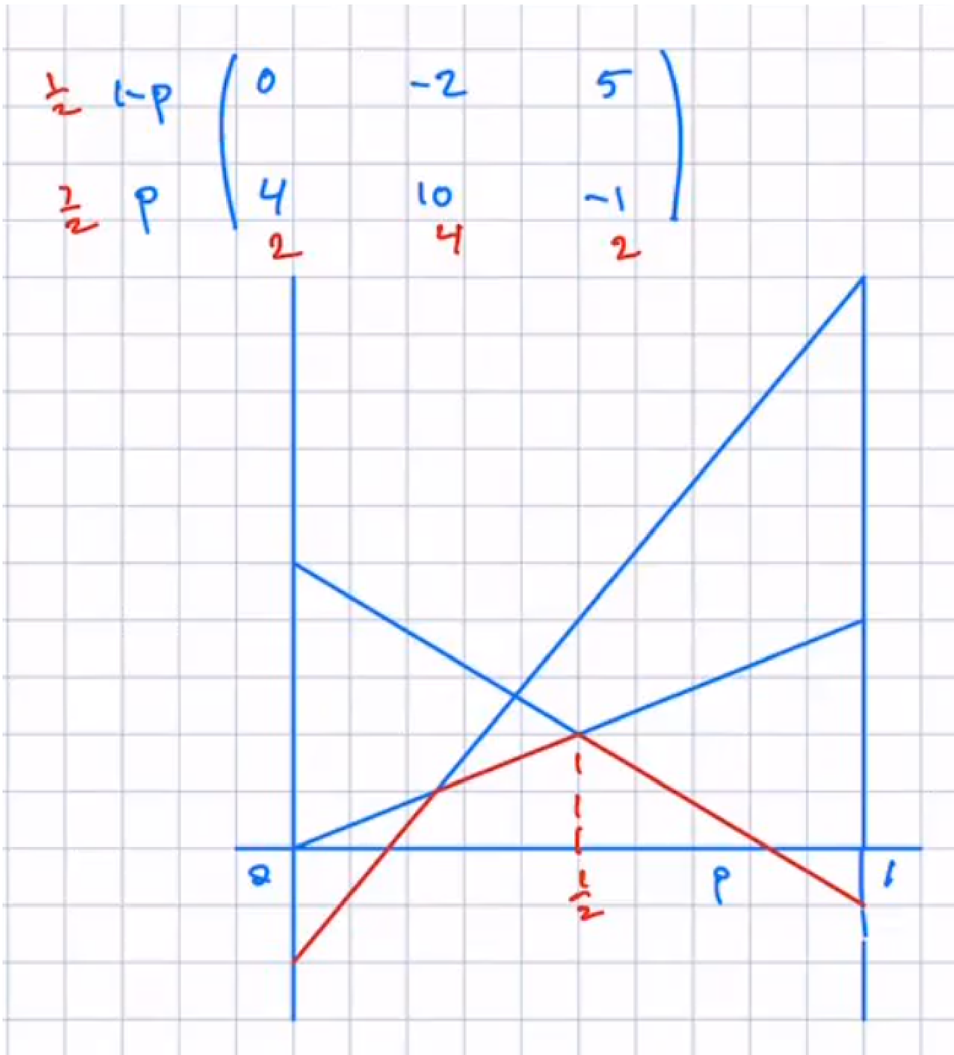
2.5 is the value of the game

The red line in the left image signifies what player 1 would try to calculate, the maximum point below all graphs. The red line in the right image is what player 2 would calculate, the minimum point above all graphs.

You get the value of the game by multiplying in a similar fashion as I showed in chapter 4 of these notes.

For such a simple game for 2 players only there is no need to draw the graphs, but in games where one of the players has more actions we have to draw the graphs to eliminate some strategies that the other player would not use i.e. the ones we need to use to calculate p or q. Example:

In the example below we drew the diagram from perspective of player 1. We see that the maximum point below all graphs (remember player 1 maximises, greedy bitch, anyways I digress) so we see this point is formed by the intersection of the line from the first column and third column, therefore we find p using the first and third column in a similar way to how we did in Chapter 4. One we have found p, we calculate the expected payoff of each column. We find that there is no point in payer 2 putting any weight on the middle column since he better off with left or right, so we calculate q using left and right column. Done!




# Dominated Strategies

Sometimes either player would never choose a certain strategy because there is another strategy that will yield a better result no matter what strategy the other player would choose. Therefore the strategy that player would never choose can be eliminated to make the game simpler.

Example:

62.c

$$\begin{pmatrix} 1 & 3 & 4 & 2 \\ 2 & 2 & 4 & 3 \\ 0 & 2 & 5 & 1 \\ 0 & 0 & 5 & 3 \end{pmatrix}$$

dominated by column 1

62.c

$$\begin{pmatrix} 1 & 3 & 4 & 2 \\ 2 & 2 & 4 & 3 \\ 0 & 2 & 5 & 1 \\ 0 & 0 & 5 & 3 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

8 dominated by row 1

2 column 4 is dominated by column 1

1 column 3 is dominated by column 1

The small numbers 1, 2, 3 show the order the columns were deleted in. The top left corner lines were last.

# Fictitious Play

Kind of simulating the game, make each player pick best reply infinitely many times and the mixed strategy is the probability of each action being taken.

| *stage* | *obs. P.1* | *obs. P.2* | *action P.1* | *action P.2* |
|---|---|---|---|---|
| 1 | *none* | *none* | 1 | 1 |
| 2 | $(1, 0, 0, 0)$ | $(1, 0)$ | 4 | 1 |
| 3 | $(0.5, 0, 0, 0.5)$ | $(1, 0)$ | 4 | 2 |
| 4 | $(0.33, 0, 0, 0.67)$ | $(0.67, 0.33)$ | | |

Fictitious play only works for zero sum games, Shapley proved this in the 70s, for example try:

$$\begin{pmatrix} 1,0 & 0,0 & 0,1 \\ 0,1 & 1,0 & 0,0 \\ 0,0 & 0,1 & 1,0 \end{pmatrix}$$